FUNDAMENTALS OF ELECTRIC CIRCUITS

Part 1: DC CIRCUITS

Chapter 2: Basic laws

I. Introduction.

II. Ohm's law.

III. Nodes, branches and loops.

IV. Kirchhoff's laws.

V. Series resistors and voltage division.

VI. Parallel resistors and current division.

VII. Wye – Delta transformations

I. Introduction

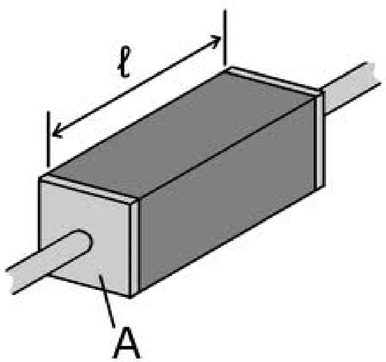
- In order to determine the values of current, voltage, and power in an electric circuit, we should understand some fundamental laws.

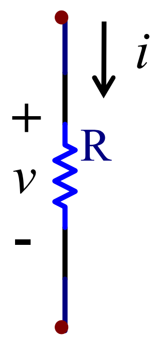
- This chapter presents:
  - Ohm’s law, Kirchhoff’s laws
  - Some techniques commonly applied in circuit design and analysis:
    - Combining resistors in series and parallel
    - Voltage division
    - Current division
    - Delta – Wye and Wye – Delta transformations

II. Ohm's law

- In general, material have a characteristic behavior of resisting the flow of electric charge.

- *Resistance* (*R*) is known as the ability to resist current.





$$\boxed{R = \rho \frac{l}{A}}$$

ρ: resistivity of the material [Ωm]

l : length of material [m]

A: cross sectional area [m^2]

- ***Ohm's law:*** The voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$\boxed{v = Ri}$$

- The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in Ohms [Ω]

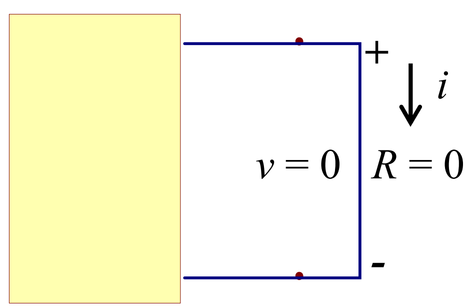
$$\boxed{R = \frac{v}{i} \rightarrow 1\Omega = 1\frac{V}{A}}$$

II. Ohm's law

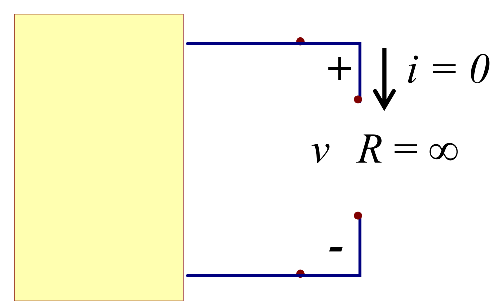
➢ There are two extreme possible values of *R*

❖ *Short circuit:* is a circuit element with resistance approaching zero (current could be anything).

❖ *Open circuit:* is a circuit element with resistance approaching infinity (voltage could be anything).



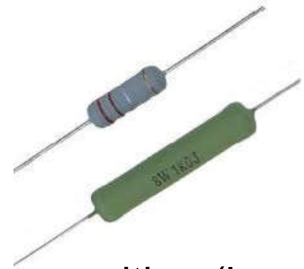
$$R=0 \rightarrow v=iR=0$$



$$i=\lim_{R\to\infty}\frac{v}{R}=0$$

II. Ohm's law

- Resister classification:
  - Fixed resistors:

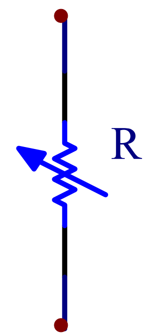


Wire wound
(small resistance

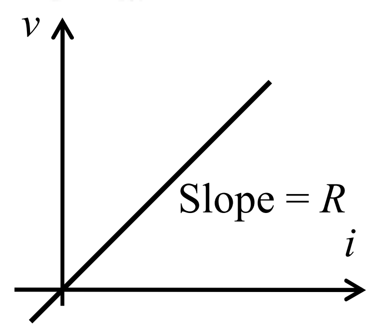
Composition (large
resistance)

Symbol for
fixed resistor

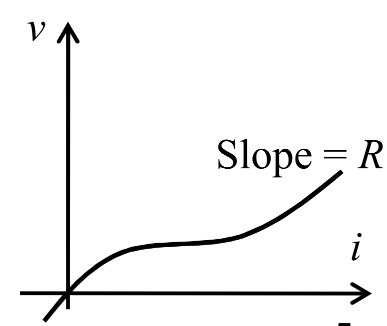
  - Variable resistors:



  - Linear resistor:



  - Nonlinear resistor: Do not consider



II. Ohm's law

- *Conductance* is the ability of an element to conduct electric current, it is measured in Siemens [S]

$$\boxed{G = \frac{1}{R} = \frac{i}{v} \qquad 1S = 1\frac{A}{V}}$$

- Power *dissipated by a resistor (conductance):*

$$\boxed{p = vi = i^2 R = \frac{v^2}{R}} \qquad \boxed{p = vi = v^2 G = \frac{i^2}{G}}$$

→ a resistor always absorbs power from the circuit (*passive element*)

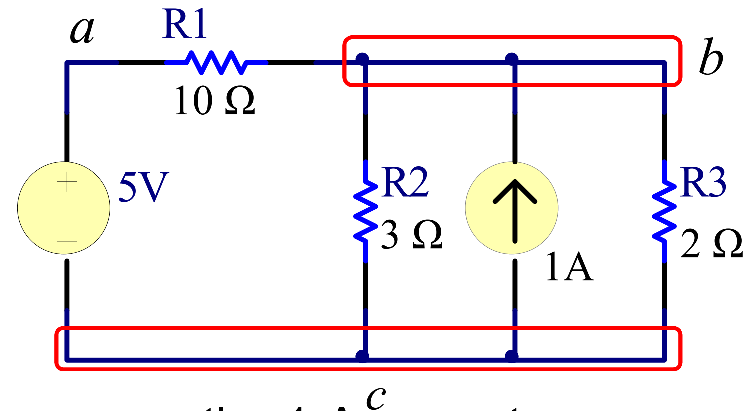
III. Nodes, branches, and loops

- Since the elements of an electric circuit can be interconnected in several ways → we need to understand some basic concepts of network topology.

- We regard a network as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths

- We study the ***properties relating to the placement of element*** in the network and the ***geometric configuration of the network***.

III. Nodes, branches, and loops



- A ***branch*** *(b)* represents a single element such as a voltage source or a resistor (represent any two terminal element).

  Ex: The circuit has five branches: the 5-V voltage source, the 1-A current source, and three resistors.

- A ***node*** *(n)* is the point of connection between two or more branches.

  Ex: The circuit has three nodes: *a, b,* and *c.*

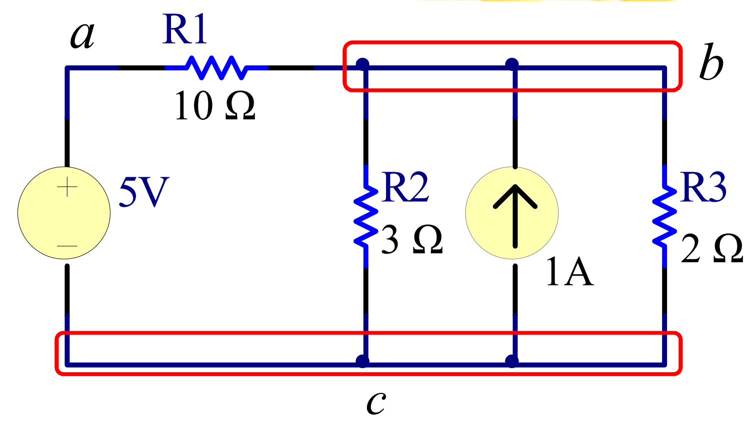
- A ***loop*** is any closed path in a circuit, formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.

  Ex: abca is a loop, containing the R1, R2 and voltage source.

III. Nodes, branches, and loops

- A loop is said to be ***independent*** if it contains a branch which is not in any other loop.

  Ex: The circuit has totally six loops, but only three of them are independent.



- A network with *b* branches, *n* nodes, and *l* independent loops, has an equation:

$$\boxed{b = l + n - 1}$$

- Two or more elements are in ***series*** if they are cascaded or connected sequentially and consequently carry the same current.

- Two or more elements are in ***parallel*** if they are connected to the same two nodes and sequentially have the same voltage across them.

IV. Kirchhoff's laws

- Kirchhoff's laws include: Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

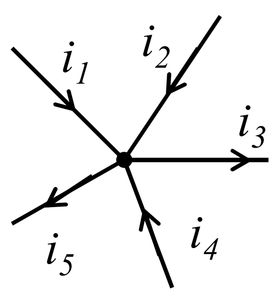
- Kirchhoff's laws, coupled with Ohm's law, make a sufficient and powerful set of tools for analyzing a large variety of electric circuits.

- ***Kirchhoff's current law (KCL)*** states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

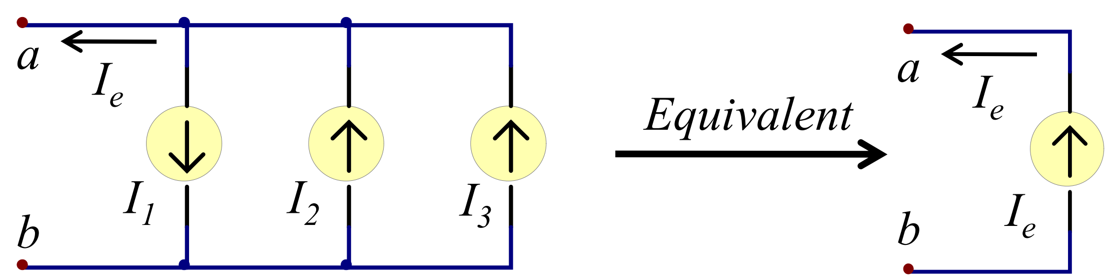
$$\boxed{\sum_{n=1}^{N} i_n = 0}$$

Convention:

- Current entering a node may be regarded as positive.
- Current leaving the node may be taken as negative.



$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$



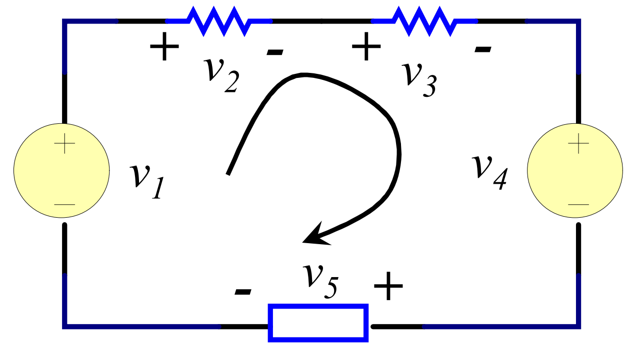
$$I_e = -I_1 + I_2 + I_3$$

IV. Kirchhoff's laws

- ***Kirchhoff's voltage law (KVL)*** states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\boxed{\sum_{m=1}^{M} v_m = 0}$$

Ex1: Write KVL for this circuit.

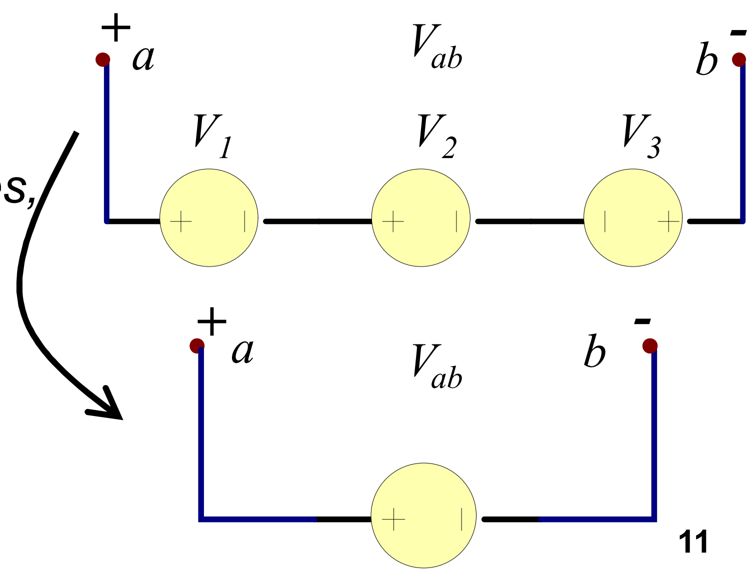


- ❖ Start with any branch and go around the loop either clockwise or counterclockwise

$$-v_1 + v_2 + v_3 + v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 - v_4$$

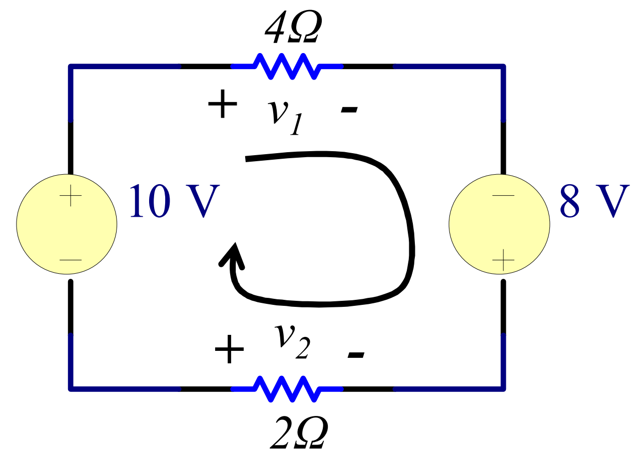
Ex2: When voltage sources are connected in series, KVL can be applied to obtain the total voltage.



$$V_{ab} = V_1 + V_2 - V_3$$

IV. Kirchhoff's laws

Ex3: Find v1 and v2 in the circuit.

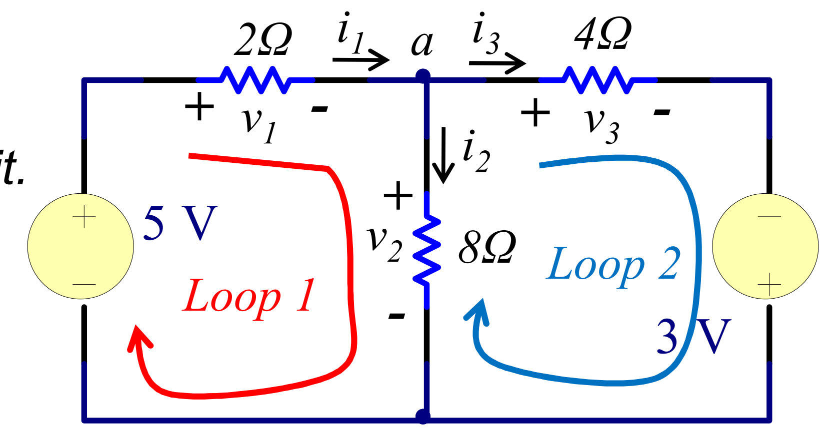


- Assume that current I flows through the loop as indicating in the Figure.
- From Ohm's law: $v_1 = 4i \quad ; \quad v_2 = -2i$
- Applying KVL around the loop gives: $v_1 - v_2 = 10 + 8 = 18$
- Substituting i in Ohm's law to KVL: $6i = 18 \rightarrow i = 3A$
- Finally we have: $v_1 = 4i = 12V$

$$v_2 = -2i = -6V$$

IV. Kirchhoff's laws

Ex4: Find the currents and voltages in the circuit.



- From Ohm's law:

$$v_1 = 2i_1 \quad ; \quad v_2 = 8i_2 \quad ; \quad v_3 = 4i_3$$

- At node *a*, applying KCL gives: $i_1 - i_2 - i_3 = 0$

- Applying KVL to loop 1 and loop 2: $\begin{cases} v_1 + v_2 = 5 \\ -v_2 + v_3 = 3 \end{cases} \rightarrow \begin{cases} 2i_1 + 8i_2 = 5 \\ -8i_2 + 4i_3 = 3 \end{cases}$

- Finally we have: $\begin{cases} i_1 - i_2 - i_3 = 0 \\ 2i_1 + 8i_2 = 5 \\ -8i_2 + 4i_3 = 3 \end{cases} \rightarrow \begin{cases} i_1 = 1,5A \\ i_2 = 0,25A \\ i_3 = 1,25A \end{cases} \rightarrow \begin{cases} v_1 = 3V \\ v_2 = 2V \\ v_3 = 5V \end{cases}$

V. Series resistors and voltage division

- The ***equivalent resistance*** of any number of resistors connected in series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n$$

- ***Voltage divider***: The voltage v is divided among the resistors in direct proportion to their resistances, the larger the resistance, the larger the voltage drop.

$$v_n = \frac{R_n}{R_1 + R_2 + \ldots + R_N} v$$

VI. Parallel resistors and current division

- The ***equivalent resistance*** of two parallel resistors is equal to the product of their resistances divided by their sum

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}$$

- The ***equivalent conductance*** of resistors connected in parallel is the sum of their individual conductances.

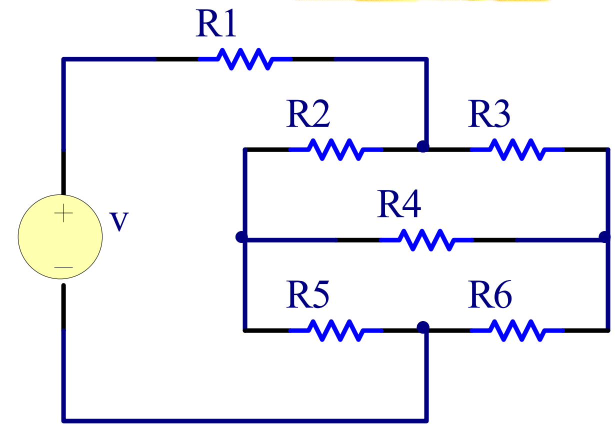
$$G_{eq} = G_1 + G_2 + \dots + G_N = \sum_{n=1}^{N} G_n$$

- ***Current divider:***

$$i_n = \frac{G_n}{G_1 + G_2 + \dots + G_N} i$$

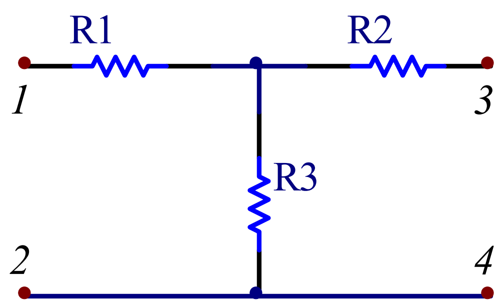
VII. Wye – Delta transformations

- How do we combine resistors when they are neither in series nor in parallel ?

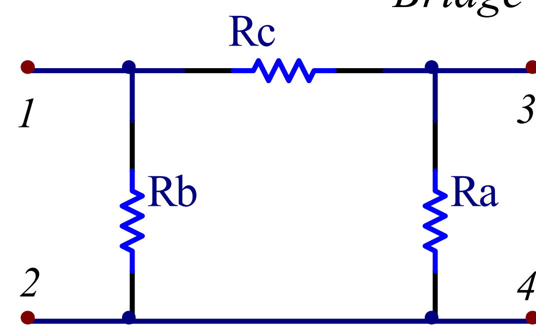


Bridge circuit

- ***Delta to Wye conversion:***



Wye (Y) or T network

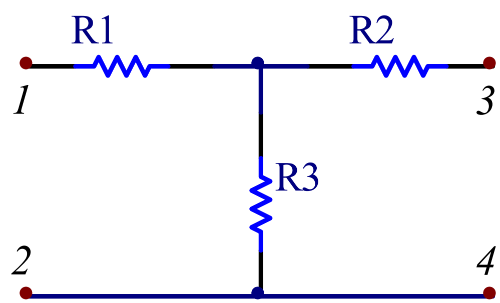


Delta (Δ) or Π network

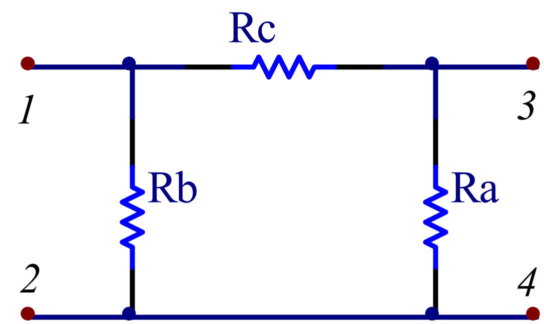
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad ; \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad ; \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

VII. Wye – Delta transformations

➢ ***Wye to Delta conversion:***



Wye (Y) or T network



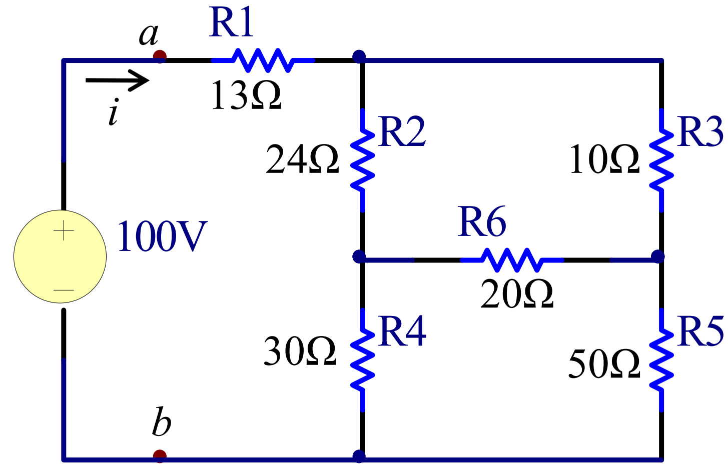
Delta (Δ) or Π network

$$R_a = R_2 + R_3 + \frac{R_2R_3}{R_1} \quad ; \quad R_b = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

$$R_c = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

VII. Wye – Delta transformations

Ex: For the bridge circuit, find R_{eq} and i

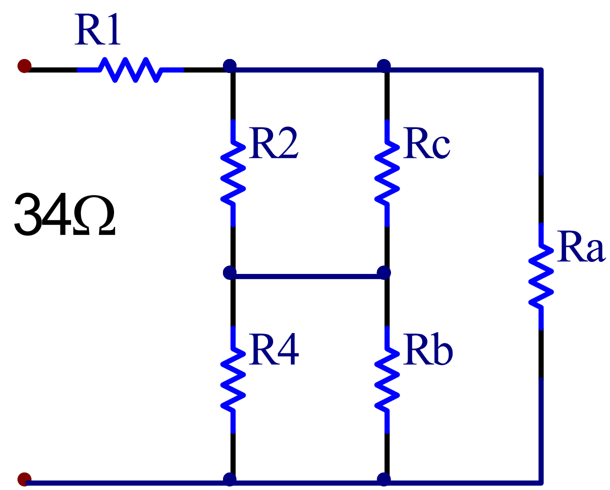


- In this circuit, there are two Y networks: (R_2, R_4, R_6) and (R_3, R_5, R_6) → transforming just one of them will simplify the circuit
- Applying the Y to Δ transformation:

$$R_a = R_3 + R_5 + \frac{R_3 R_5}{R_6} = 85\Omega$$

$$R_c = R_3 + R_6 + \frac{R_3 R_6}{R_5} = 34\Omega$$

$$R_b = R_5 + R_6 + \frac{R_5 R_6}{R_3} = 170\Omega$$



- Combining all resistors, we obtain:

$$R_{eq} = R_1 + \left\{ \left[\left(R_2 // R_c \right) + \left(R_4 // R_b \right) \right] // R_a \right\}$$

$$R_{eq} = 40\Omega$$

$$\rightarrow i = \frac{u_{ab}}{R_{eq}} = \frac{100}{40} = 2,5A$$