Chapter 5: Operational amplifiers

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Chapter 5: Operational amplifiers

I. Introduction

- The operational amplifier (*op amp*) is an electronic unit that behaves like a voltage-controlled voltage source.
- An op amp can: sum signals, amplify, integrate, or differentiate a signal.
- Nowadays, op amp are popular in practical circuit designs because they are versatile, inexpensive, easy to use …
- This chapter presents the ideal op amp first and consider the non-ideal op amp later.
- Using nodal analysis as a tool, we consider ideal op amp circuits such as the inverter, voltage follower, summer and difference amplifier.

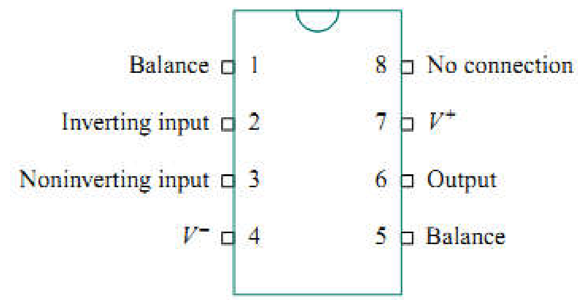
II. Operational amplifiers

- An *op amp* is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

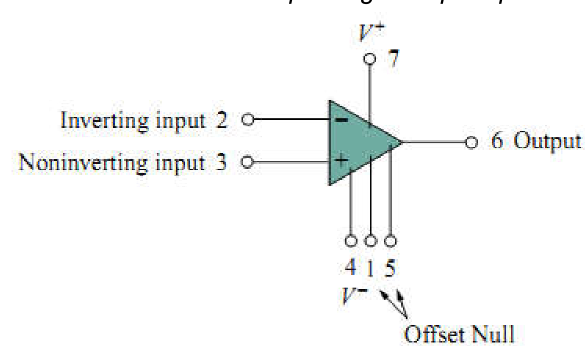
An op amp package

- The op amp is an electronic device consisting of a complex arrangement of resistor, transistor, capacitor, and diodes → treat them as a circuit building block, study what takes place at its terminals.

- There are several important terminal in an op amp:
  - The inverting input: pin 2 (-)
  - The non-inverting input: pin 3 (+)
  - The output: pin 6
  - The positive power supply V^+: pin 7
  - The negative power supply V^-: pin 4

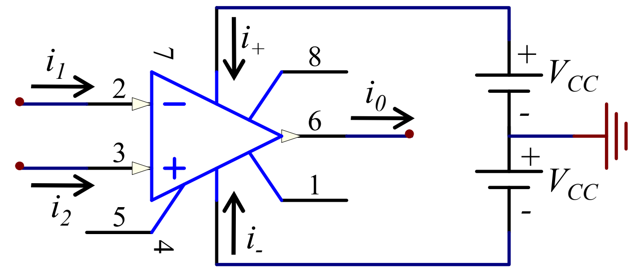


Dual in-line package of op amp



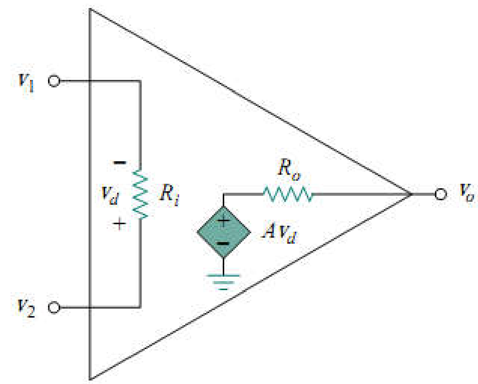
II. Operational amplifiers

- An op amp must be powered by a voltage supply which are often ignored in op amp circuit diagram, but the power supply currents must not be overlook.



$$i_0 = i_1 + i_2 + i_+ + i_-$$

- The equivalent circuit model of a non-ideal op amp:
  - The output section: Voltage-controlled source $A.v_d$ in series with output resistance R_0.
  - R_i: Thevenin equivalent resistance seen at input terminals
  - R_0: Thevenin equivalent resistance seen at the output
  - Voltage output: $v_0 = A.v_d = A.(v_2 - v_1)$



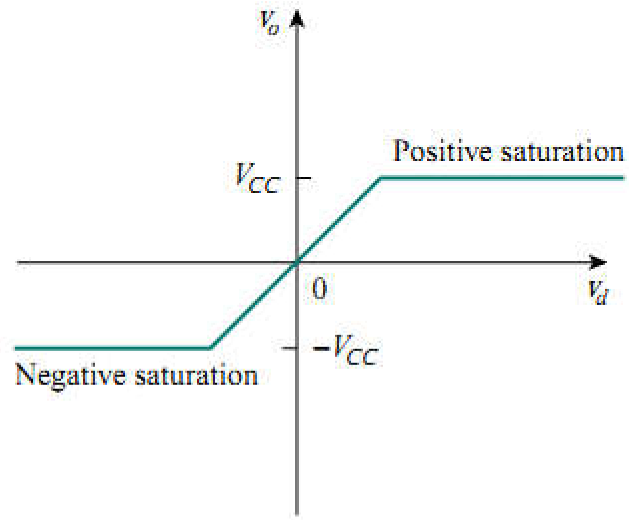
The equivalent circuit of the non-ideal op amp

A: *Open-loop voltage gain* (gain of the op amp without any external feedback from output to input.

II. Operational amplifiers

- Table shows typical parameter values of op am:

Parameter	Typical range	Ideal value
Open-loop gain, A	10^5 - 10^8	∞
Input resistance, R_i	10^6 - 10^{13} Ω	∞ Ω
Output resistance, R_0	10 - 100 Ω	0 Ω
Supply voltage, V_{cc}	5 - 24 V	



- Op amp can operate in three modes, depending on the v_d
  - Positive saturation: $v_0 = V_{cc}$
  - *Linear region:* $-V_{cc} \leq v_0 = A.v_d \leq V_{cc}$
  - Positive saturation: $v_0 = -V_{cc}$

II. Operational amplifiers

741 parameter	Value
Open-loop gain, A	2.10^5
Input resistance, R_i	$2\ M\ \Omega$
Output resistance, R_0	$50\ \Omega$

Ex 5.1: Find the closed-loop gain V_0 / V_s. Determine current i when $V_s = 2V$

➢ Using the op amp model, we obtain the equivalent circuit → applying the nodal analysis gives:

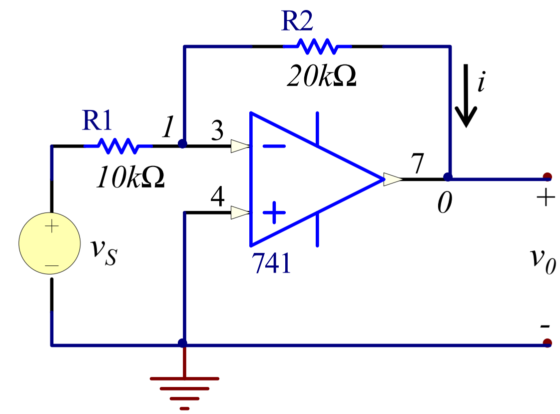
❖ At node 1: $$\frac{V_S - V_1}{R_1} = \frac{V_1}{R_i} + \frac{V_1 - V_0}{R_2} \rightarrow V_1 = \frac{2V_S + V_0}{3}$$

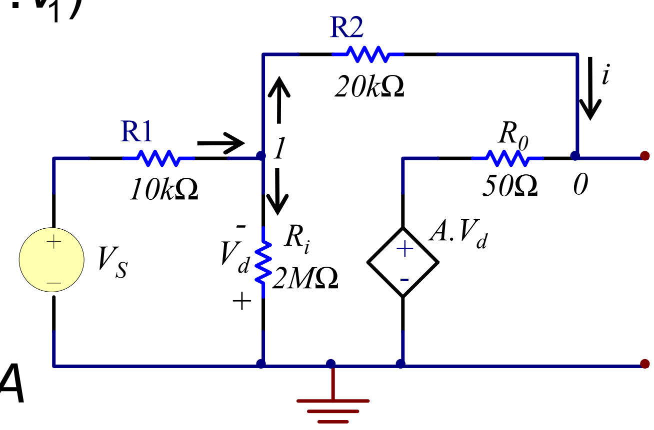
❖ At node 0:

$$\begin{cases} \dfrac{V_1 - V_0}{R_2} = \dfrac{V_0 - AV_d}{R_0} & \rightarrow V_1 - V_0 = 400(V_0 + 2.10^6.V_1) \\ V_d = -V_1 & \rightarrow \dfrac{V_0}{V_s} \simeq -1.9999699 \end{cases}$$

❖ When $v_s = 2V$: $$V_1 = \frac{2V_S + V_0}{3} = 20{,}067\mu V$$

$$\rightarrow i = \frac{V_1 - V_0}{R_2} \simeq 0{,}2mA$$

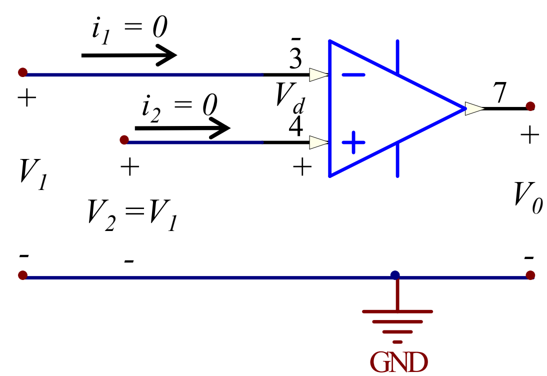




III. Ideal op amp

- An *ideal op amp* is an amplifier with infinite open-loop gain ($A = \infty$), infinite input resistance ($R_i = \infty$), and zero output resistance ($R_O = 0$).

- Important characteristics of an ideal op amp:



  - The currents into both input terminal are zero

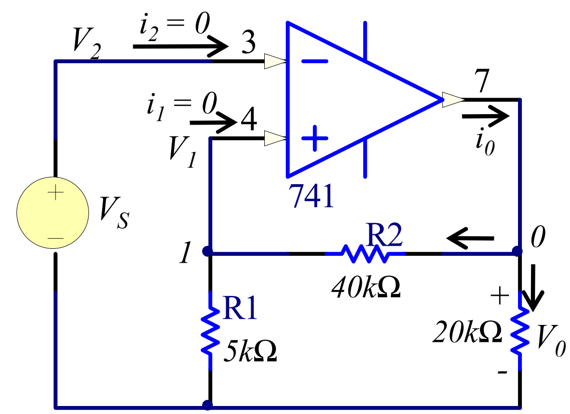
$$\boxed{i_1 = 0 \quad ; \quad i_2 = 0}$$

  - The voltage across the input terminals is small

$$\boxed{V_d = V_2 - V_1 \simeq 0 \quad ; \quad V_1 = V_2}$$

III. Ideal op amp

Ex 5.2: Considering 741 op amp as an ideal op amp. Calculate the V_0/V_s. Find i_0 when $v_s = 1V$



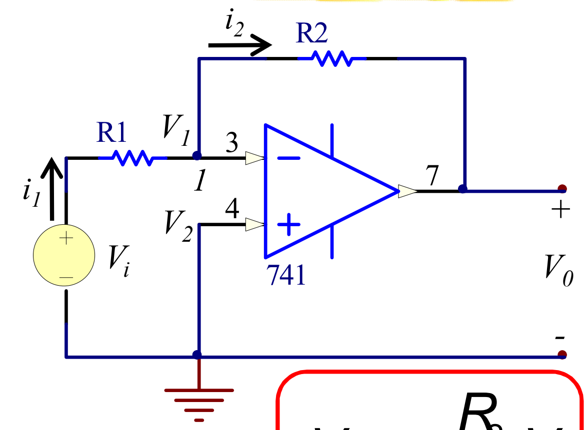
- As an ideal op amp, we have: $V_2 = V_S$
- Since $i_1 = 0$ → R_1 and R_2 are in series

$$V_1 = \frac{V_0}{R_1 + R_2} R_1 = \frac{V_0}{9}$$

- From these equations, we have: $V_S = \frac{V_0}{9} \rightarrow \frac{V_0}{V_S} = 9$
- Applying KCL at node 0 gives: $i_0 = \frac{V_0}{R_1 + R_2} + \frac{V_0}{20.10^3} \xrightarrow{V_S=1} \begin{cases} V_0 = 9V \\ i_0 = 0,65mA \end{cases}$

IV. Inverting – Non-inverting amplifier

IV.1. Inverting amplifier



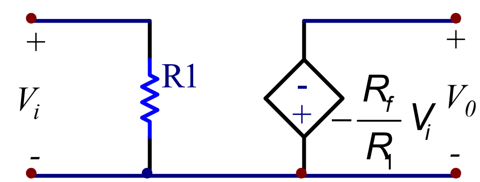
- Applying KCL at node 1:

$$i_1 = i_2 \rightarrow \frac{V_i - V_1}{R_1} = \frac{V_1 - V_0}{R_2} \rightarrow \frac{V_i}{R_1} = -\frac{V_0}{R_2} \rightarrow V_0 = -\frac{R_2}{R_1} V_i$$

- Ideal op amp: $V_1 = V_2 = 0$

$$\boxed{V_0 = -\frac{R_2}{R_1} V_i}$$

- **Notes:**
  - The voltage gain $A_V = -\frac{R_2}{R_1}$ depends only on the external element connected to the op amp.
  - A key feature of the inverting amplifier is that both the input signal and the feedback are applied at the inverting terminal of the op amp.

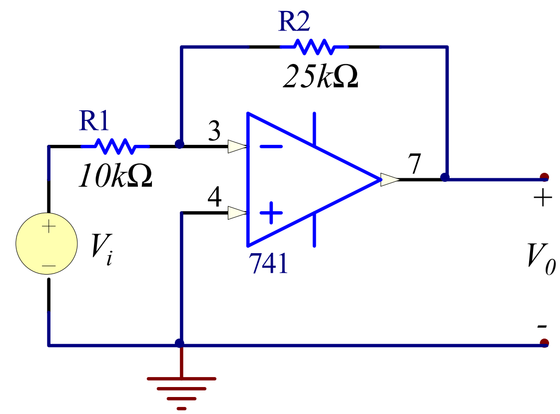


  - The inverting amplifier is used in a current to voltage converter.

IV. Inverting – Non-inverting amplifier

IV.1. Inverting amplifier

Ex 5.3: Calculate the output voltage v_0 and the current through R_1 and R_2 if $v_i = 0,5V$



- This is an inverting amplifier

$$V_0 = -\frac{R_2}{R_1}.V_i = -\frac{25}{10}.0,5 = -1,25V$$

- Calculating the current though the R_1

$$i_{R1} = \frac{V_i - 0}{R_1} = \frac{0,5}{10.10^3} = 50\mu A$$

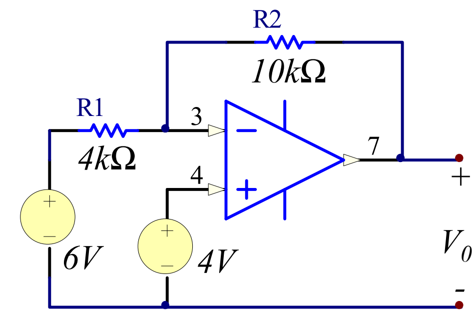
- Calculating the current though the R_2

$$i_{R2} = \frac{V_0 - 0}{R_2} = \frac{-1.25}{25.10^3} = -50\mu A$$

IV. Inverting – Non-inverting amplifier

IV.1. Inverting amplifier

Ex 5.4: Calculate the output voltage V_0



- Applying KCL at node 3:

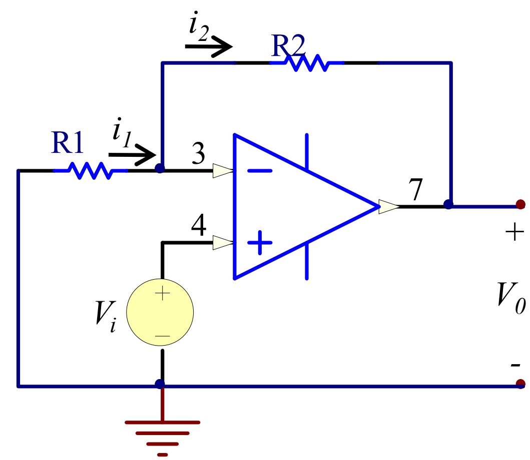
$$\frac{6-V_3}{R_1}=\frac{V_3-V_O}{R_2}\rightarrow\frac{6-V_3}{4}=\frac{V_3-V_O}{10}$$

- Because of an ideal op amp, we have $V_3 = V_4 = 4V$

$$\rightarrow\frac{6-4}{4}=\frac{4-V_O}{10}\rightarrow V_O=-1V$$

IV. Inverting – Non-inverting amplifier

IV.2. Non-inverting amplifier



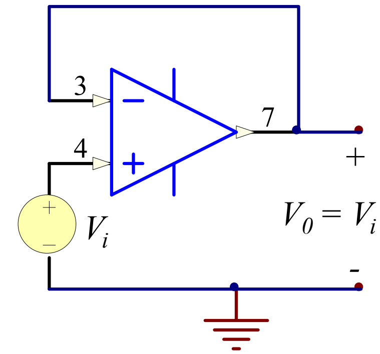
- A *non-inverting amplifier* is an op amp circuit designed to provide a positive voltage gain:
  - Input voltage V_i is applied directly at the non-inverting input terminal.
  - R1 is connected between the ground and the inverting terminal.

$$i_1 = i_2 \rightarrow \frac{0 - V_3}{R_1} = \frac{V_3 - V_0}{R_2} \rightarrow \frac{-V_i}{R_1} = \frac{V_i - V_0}{R_2} \rightarrow \boxed{V_0 = \left(1 + \frac{R_2}{R_1}\right) V_i}$$

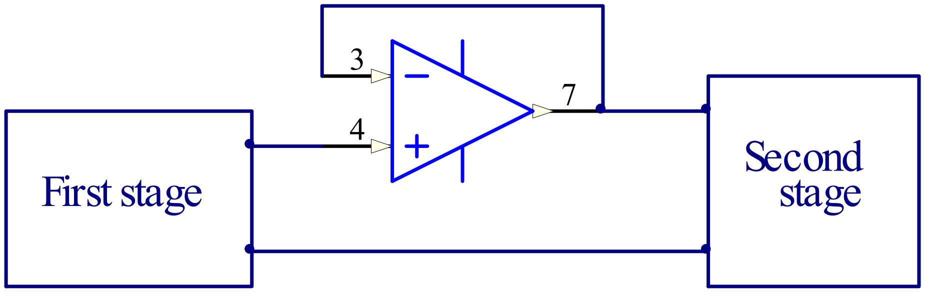
$$V_3 = V_4 = V_i$$

IV. Inverting – Non-inverting amplifier

IV.2. Non-inverting amplifier



- ***Note:***
  - If $R_2 = 0$ or $R_1 = \infty \rightarrow$ the voltage gain becomes 1
  - In this case, the circuit is call a *voltage follower* (or *unity gain amplifier*)
- Characteristic of voltage follower:
  - A very high input impedance
  - Useful as an intermediate - stage (or *buffer*) amplifier to isolate one circuit from another.



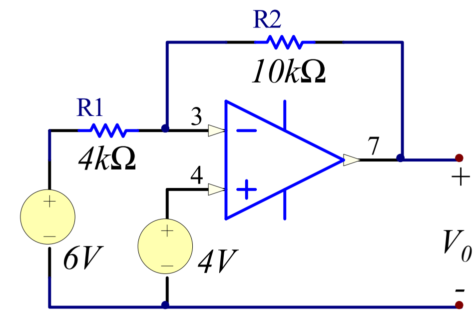
Voltage follower used to isolate two cascaded stages of a circuit

  - Minimize interaction between the two stages and eliminate inter-stage loading.

IV. Inverting – Non-inverting amplifier

IV.2. Non-inverting amplifier

Ex 5.5: Calculate the output voltage V_0



- Using superposition lets: $V_O = V_{O1} + V_{O2}$
  - V_{O1} is due to the 6V voltage source
  - V_{O2} is due to the 4V voltage source
- Calculate V_{O1}: Set the 4V voltage source to zero, the circuit becomes an inverter.

$$V_{O1} = -\frac{R_2}{R_1}6 = -\frac{10}{4}6 = -15V$$

- Calculate V_{O2}: Set the 6V voltage source to zero, the circuit becomes an non-inverter amplifier

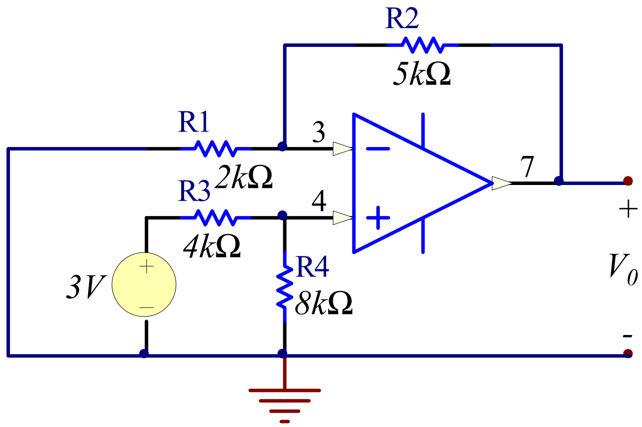
$$V_{O2} = \left(1 + \frac{R_2}{R_1}\right)4 = \left(1 + \frac{10}{4}\right)4 = 14V$$

$$\rightarrow V_O = V_{O1} + V_{O2} = -1V$$

IV. Inverting – Non-inverting amplifier

IV.2. Non-inverting amplifier

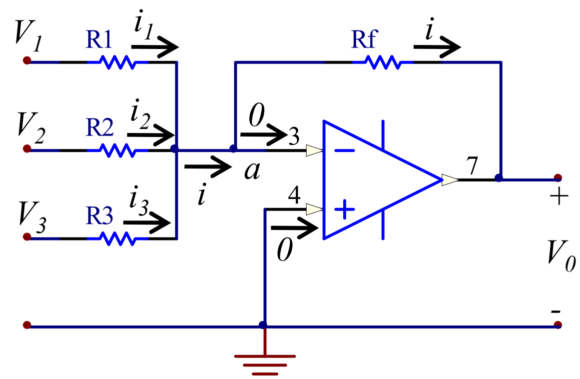
Ex 5.6: Calculate the output voltage V_0



V. Summing amplifier

- A *summing amplifier* is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

- The summing amplifier is a variation of the inverting amplifier



- Applying KCL at node *a* gives:

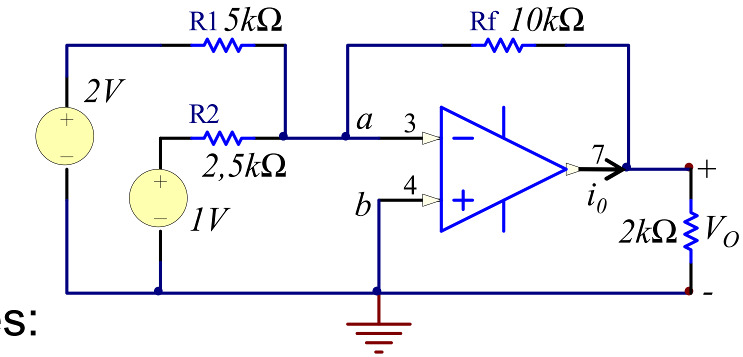
$$i = i_1 + i_2 + i_3 \rightarrow \frac{V_a - V_O}{R_f} = \frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} + \frac{V_3 - V_a}{R_3}$$

- Note that: $V_a = 0$ (ideal op amp)

$$\rightarrow \boxed{V_O = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right)}$$

V. Summing amplifier

Ex 5.7: Calculate V_0 and i_0 in the op amp circuit



- This is a summer with two inputs
- Applying the equation of summing amplifier gives:

$$V_O = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right) = -\left(\frac{10}{5}.2 + \frac{10}{2,5}\right) = -8V$$

- The current i_0 is the sum of the currents through the 10-kΩ and 2-kΩ resistor

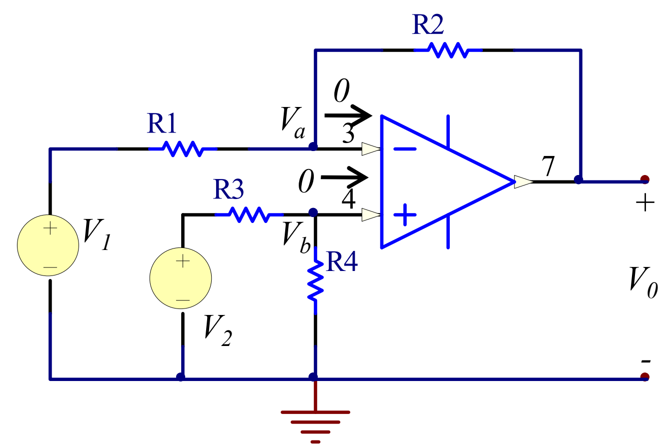
$$i_0 = \frac{V_O - V_a}{10} + \frac{V_O - V_b}{2} = \frac{-8}{10} + \frac{-8}{2} = -4,8mA$$

VI. Difference amplifier

- A *difference (differential) amplifier* is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

- Applying KCL at node *a* gives:

$$\frac{V_1 - V_a}{R_1} = \frac{V_a - V_0}{R_2} \rightarrow V_0 = \left(\frac{R_2}{R_1} + 1\right) V_a - \frac{R_2}{R_1} V_1$$



- Applying KCL at node *b* gives:

$$\frac{V_2 - V_b}{R_3} = \frac{V_b}{R_4} \rightarrow V_b = \frac{R_4}{R_3 + R_4} V_2$$

- Note that: $V_a = V_b$ (ideal op amp):

$$V_0 = \left(\frac{R_2}{R_1} + 1\right) \frac{R_3}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1 \rightarrow \boxed{V_0 = \frac{R_2}{R_1} \frac{\left(1 + \frac{R_1}{R_2}\right)}{\left(1 + \frac{R_3}{R_4}\right)} V_2 - \frac{R_2}{R_1} V_1}$$

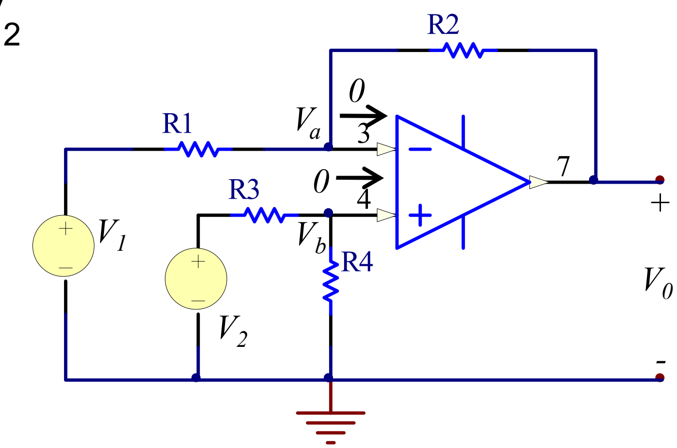
VI. Difference amplifier

- Since a difference apmlifier must reject a signal common to the two inputs

$$V_0 = 0 \text{ when } V_1 = V_2$$

- This properties exists when: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

- The op amp circuit is a difference amplifier



$$\rightarrow V_0 = \frac{R_2}{R_1}\left(V_2 - V_1\right)$$

- If $R_2 = R_1$, and $R_3 = R_4$, the difference amplifier becomes a subtractor $V_0 = V_2 - V_1$

- Remarks:
  - The difference amplifier is also known as the subtractor
  - The difference amplifier are used in varios applications (*instrumentation amplifier*)

VI. Difference amplifier

Ex 5.8: Design an op amp circuit with inputs V_1 and V_2 such that $V_0 = -5V_1 + 3V_2$

- *Solution 1:* Using only one op amp

  - Rewrite: $V_0 = -5V_1 + 3V_2 = 5\left(\frac{3}{5}V_2 - V_1\right)$

  - Applying the difference amplifier equation: $V_0 = \frac{R_2}{R_1}(V_2 - V_1) \to \boxed{\frac{R_2}{R_1} = 5}$

  - In the other word: $V_0 = \frac{R_2}{R_1}\frac{\left(1+\frac{R_1}{R_2}\right)}{\left(1+\frac{R_3}{R_4}\right)}V_2 - \frac{R_2}{R_1}V_1 = 5\frac{\left(1+\frac{1}{5}\right)}{\left(1+\frac{R_3}{R_4}\right)}V_2 - 5V_1$

$$\to \frac{\left(1+\frac{1}{5}\right)}{\left(1+\frac{R_3}{R_4}\right)} = \frac{3}{5} \to 2 = 1 + \frac{R_3}{R_4} \to \boxed{R_3 = R_4}$$

  - Choose:

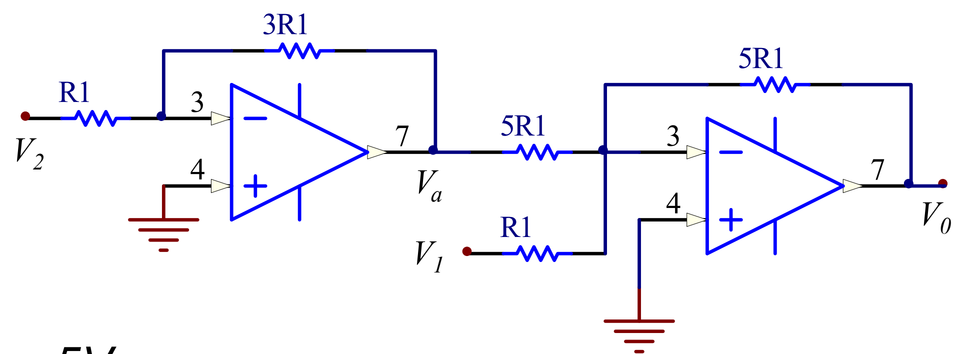
$$R_1 = 10k\Omega \; ; \; R_2 = 50k\Omega$$

$$R_3 = R_4 = 20k\Omega$$

VI. Difference amplifier

Ex 5.8: Design an op amp circuit with inputs V_1 and V_2 such that $V_0 = -5V_1 + 3V_2$

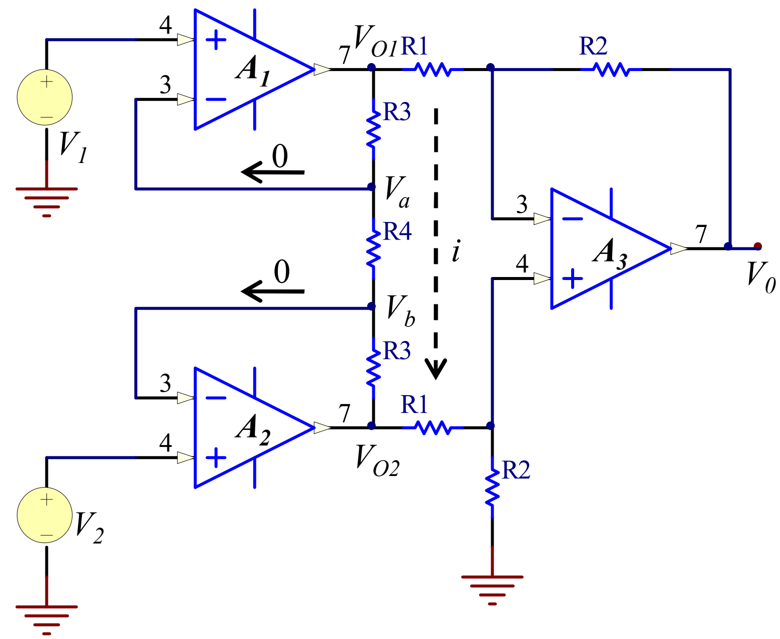
- *Solution 2:* Using > 01 op amp → inverting amplifier + 2-inputs inverting summer.



- For the summer: $V_0 = -V_a - 5V_1$
- For the inverter: $V_a = -3V_2$
- Combining 02 op amps: $V_0 = 3V_2 - 5V_1$
- Selecte the resistor value: $R_1 = 10k\Omega;$

VI. Difference amplifier

Ex 5.9: Find the relationship between V_0 and 02 - inputs of an instrumentation amplifier.



- There are not current into A_1, and A_2, the current I flows through the 3 resisters

$$V_{01} - V_{02} = i(2R_3 + R_4)$$

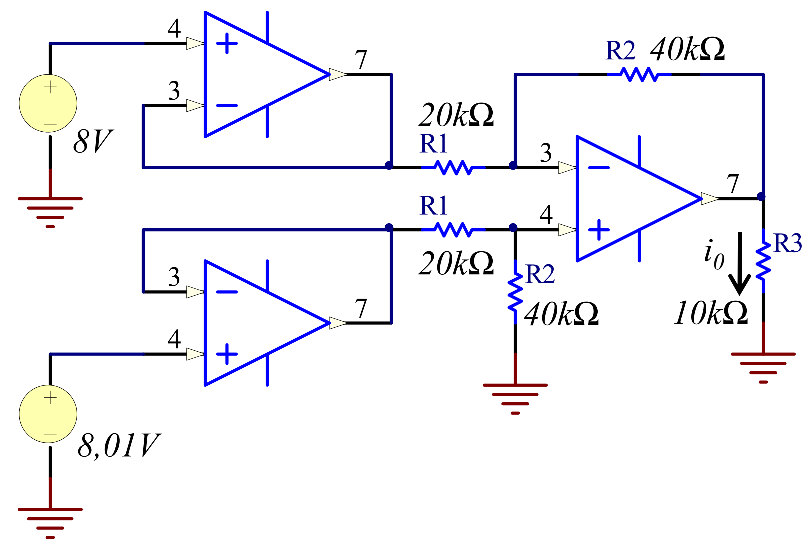
- But: $i = \dfrac{V_a - V_b}{R_4}$; $V_a = V_1$; $V_b = V_2$
- Therefore: $i = \dfrac{V_1 - V_2}{R_4}$

- The relationship between inputs and output of an intrumentation amplifier:

$$\boxed{V_0 = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)\left(V_2 - V_1\right)}$$

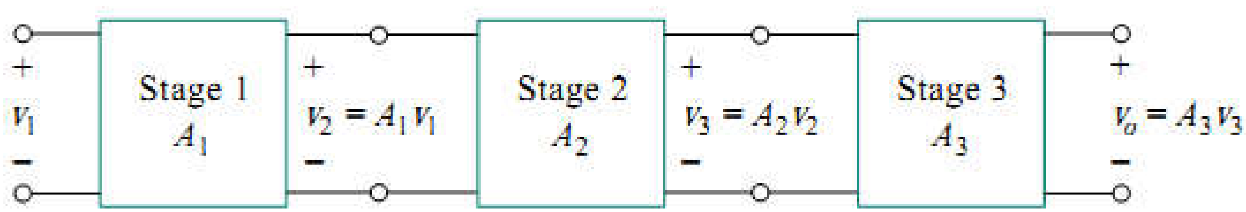
VI. Difference amplifier

Ex 5.10: Obtain i_0 in the instrumentation amplifier circuit.



VII. Cascaded op amp circuits

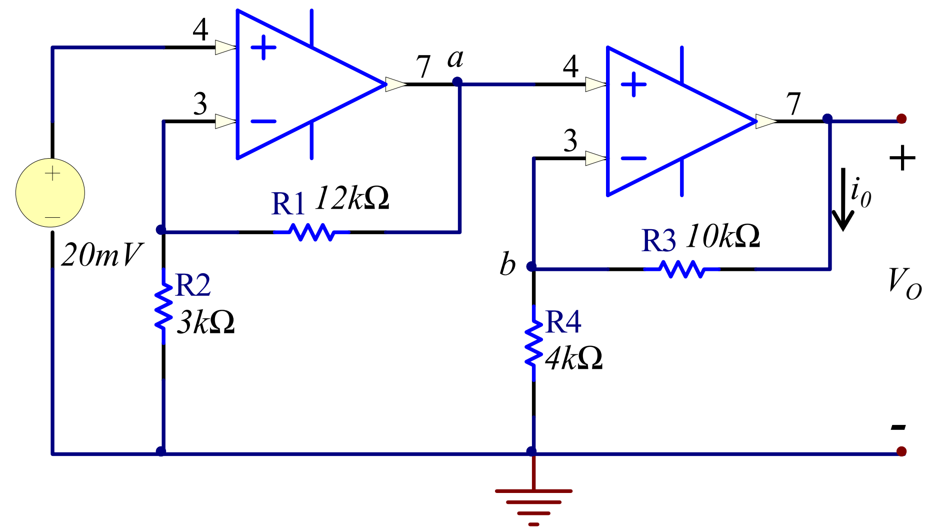
- A *cascade connection* is a head-to-tail arragement of two or more op amp circuits such that the output of one is the input of the next.
- Characteristics:
  - Each op amp circuit in the string is called a *stage.*
  - Op amp circuits can be cascaded without changing their input-output relationships beacause:
    - Infinite input resistance.
    - Zero output resistance.
  - The original input signal is increased by the gain of the individual stage.



$$\boxed{A = A_1.A_2.A_3}$$

VII. Cascaded op amp circuits

Ex. 5.11: Find V_0 and i_0 in the circuit



- The circuit consists of two non-inverting amplifiers cascaded
  - At *a* point:

$$V_a = \left(1 + \frac{R_1}{R_2}\right) 20 = 100mV$$

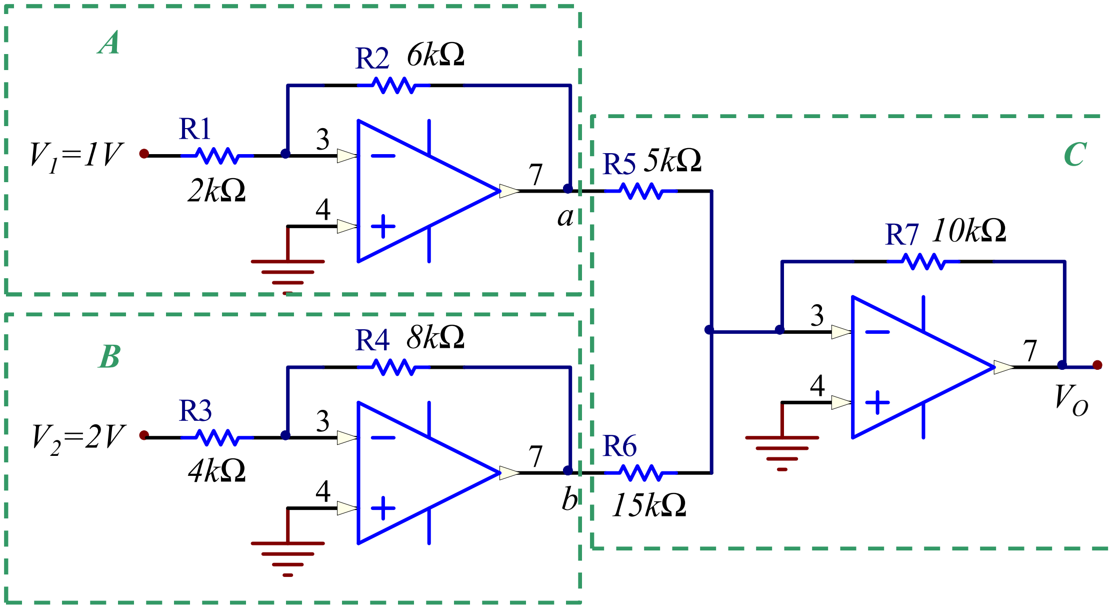
  - At the output of the second op amp: $V_O = \left(1 + \frac{R_3}{R_4}\right) V_a = 350mV$

- The current i_0 flows through the 10kΩ resistor

$$i_0 = \frac{V_O - V_b}{R_3} \xrightarrow{V_b = V_a} i_0 = \frac{(350 - 100).10^{-3}}{10.10^3} = 25\mu A$$

VII. Cascaded op amp circuits

Ex. 5.12: Find V_0



❖ The circuit consists of two inverters *A* and *B* and a summer *C*.

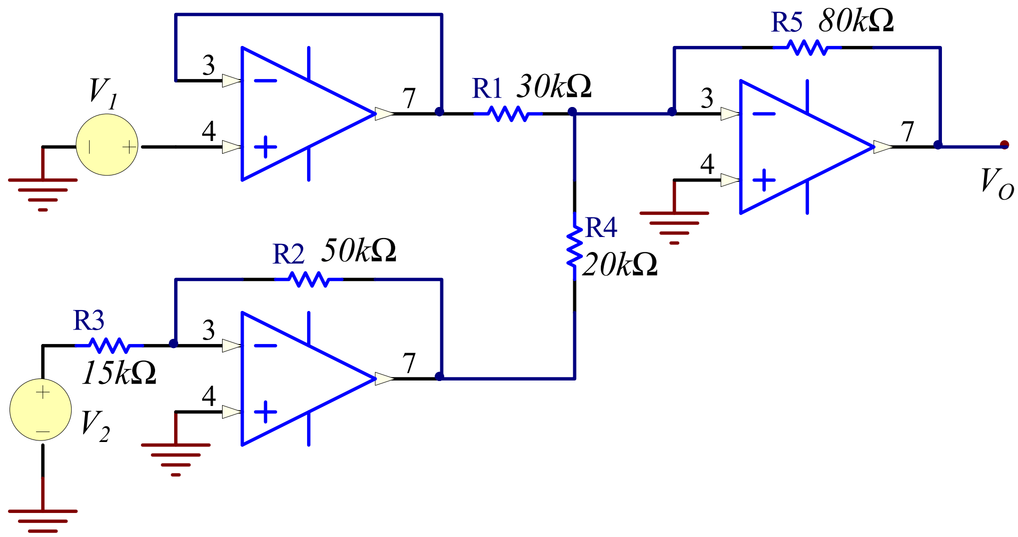
$$V_a = -\frac{R_2}{R_1}V_1 = -3V \qquad V_b = -\frac{R_4}{R_3}V_2 = -4V$$

❖ These become the inputs to the summer:

$$V_O = -\left(\frac{R_7}{R_5}V_a + \frac{R_7}{R_6}V_b\right) = -\left[2.(-3) + \frac{2}{3}(-4)\right] = 8.333V$$

VII. Cascaded op amp circuits

Ex. 5.13: Find V_0 if $V_1 = 2V$, $V_2 = 1,5V$

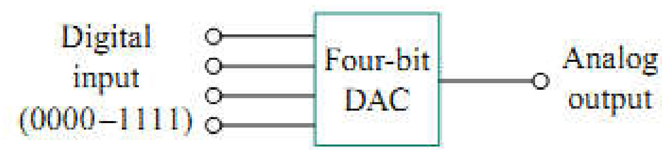


VIII. Applications

- Op amp has numerous pratical applications:
  - Inverters, summers, integrators, differentiators, subtractors, logarithmic amplifiers
  - *Instrumentation amplifiers*, calibration circuits
  - *DAC*, voltage-to-curent converters, current-to-voltage converters
  - Analog computers,
  - Filters, clippers, rectifier, regulators, level shifters
  - Comparators, gyrators, oscillators
  - …

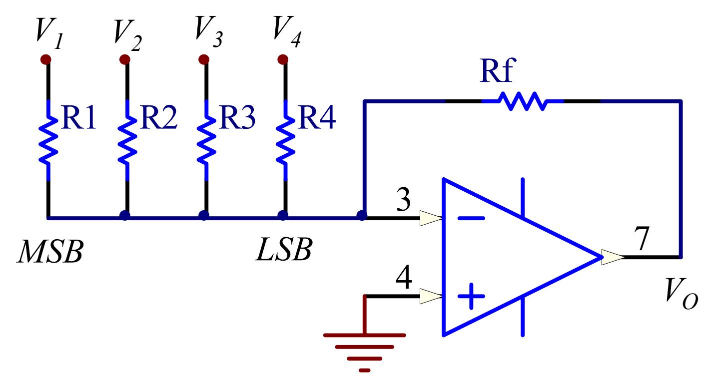
VIII.1. DAC – Digital-to-Analog Converter

- The *digital-to-analog converter* (DAC) transforms digital signals into analog form.
- A DAC can be realized by using the binary weighted ladder:



A four-bit DAC

  - The bits are weights according to the magnitude of their place value.
  - Their weights decrease value of R_f/R_n → each lesser bit has half the weight of the next higher.



Binary weighted ladder type

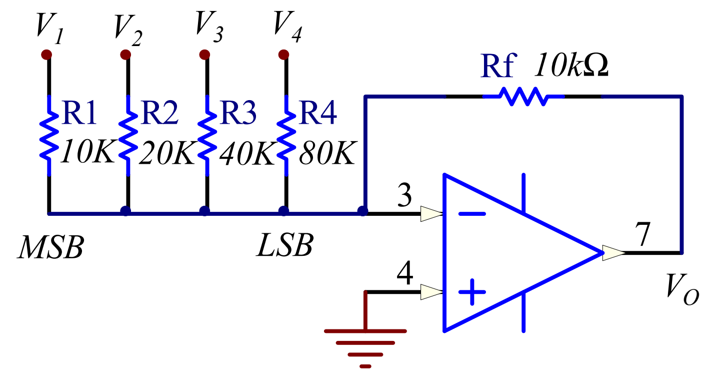
$$-V_O = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

  - V1, … V4 can assume only two voltage levels (0, 1) (*binary code*) → DAC provides a single output that is proportional to the inputs.

VIII.1. DAC – Digital-to-Analog Converter

Ex 5.14: Obtain the analog output for binary inputs [0000], [0001], [0010], … [1111].

Inputs [B]	Value [D]	$-V_0$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
…		
1111	15	1.875



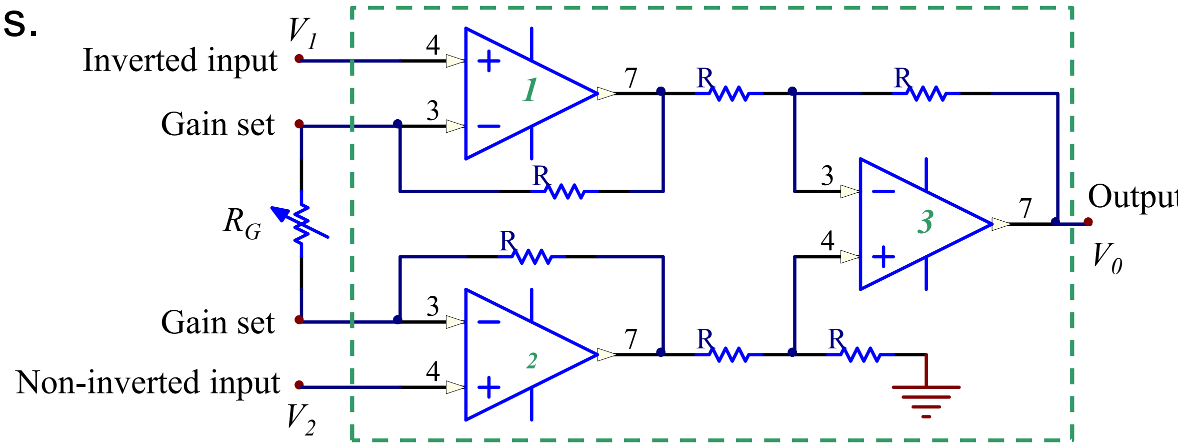
$$-V_O = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

$$-V_0 = V_1 + 0{,}5V_2 + 0{,}25V_3 + 0{,}125V_4$$

- Each bit has a value of 0.125V → cannot represent a voltage between 1V → 1.125V (*DAC resolution*).

VIII.2. Intrumentation amplifier (IA)

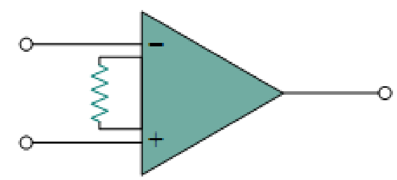
- Typical applications of IAs include isolation ampli ers, thermocouple ampli ers, and data acquisition systems.



- From the Ex 5.9, we have:

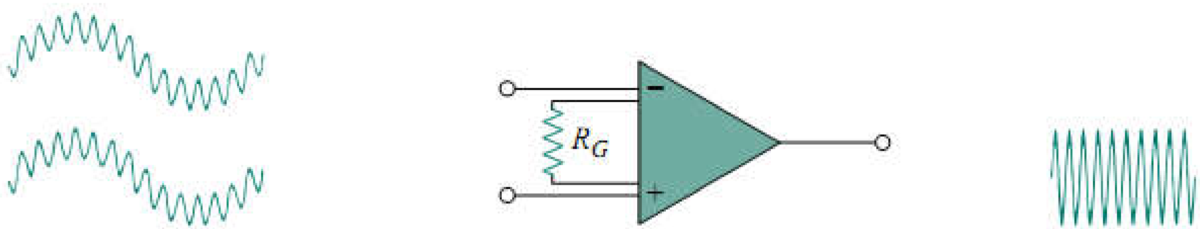
$$V_0 = \left(1 + \frac{2R}{R_G}\right)\left(V_2 - V_1\right)$$

- Recall that:
  - The IA ampli es small differential signal voltages superimposed on larger common-mode voltages.
  - Since the common-mode voltages are equal, they cancel each other.



Schematic diagram

VIII.2. Intrumentation amplifier (IA)



Small differential signals riding on larger common–mode signals — Instrumentation amplifier — Amplified differential signal, No common-mode signal

- The IAs have three major characteristics:
  - The *voltage gain is adjusted* by one externer resistor R_G
  - The *input impedance of both inputs is very high* and does *not vary* as the gain is adjusted.
  - The *output V_O depends on the difference* between the inputs V_1 and V_2, not on the voltage common to them (common-mode voltage).

VIII.2. Intrumentation amplifier (IA)

Ex: A precision Intrumentation amplifier

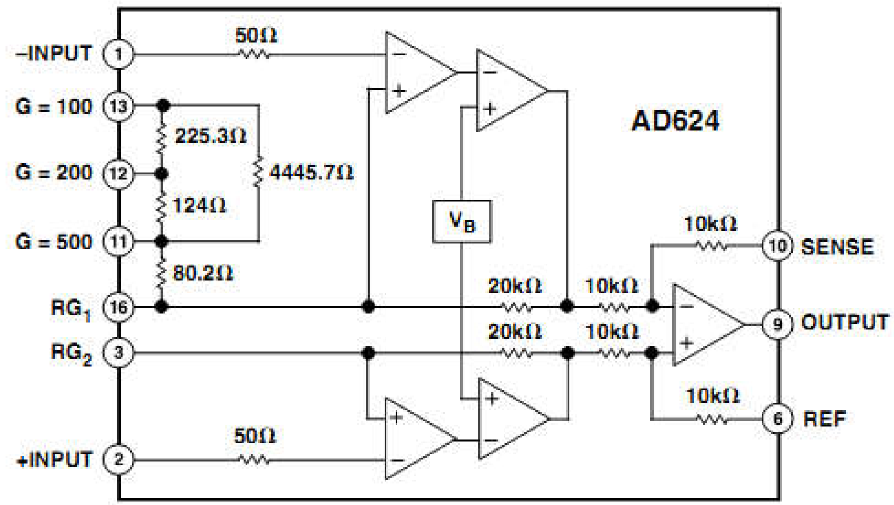
Product highlight:

- Input noise is less than 4 nV/√Hz at 1 kHz.
- Pin programmable gains of 1, 100, 200, 500 and 1000 provided on the chip. Using a single external resistor for other gains.
- The offset voltage, offset voltage drift, gain accuracy and gain temperature coefficients are guaranteed for all pretrimmed gains.
- Provides totally independent input and output offset for high precision applications.
- A sense terminal is provided to enable the user to minimize the errors induced through long leads. A reference terminal is also provided to permit level shifting at the output.

Datasheet:

http://www.analog.com/static/imported-files/data_sheets/AD620.pdf

FUNCTIONAL BLOCK DIAGRAM

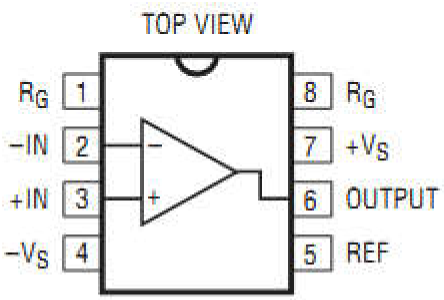


FEATURES
Low Noise: 0.2 μV p-p 0.1 Hz to 10 Hz
Low Gain TC: 5 ppm max (G = 1)
Low Nonlinearity: 0.001% max (G = 1 to 200)
High CMRR: 130 dB min (G = 500 to 1000)
Low Input Offset Voltage: 25 μV, max
Low Input Offset Voltage Drift: 0.25 μV/°C max
Gain Bandwidth Product: 25 MHz
Pin Programmable Gains of 1, 100, 200, 500, 1000
No External Components Required
Internally Compensated

Price (100 - 499)	Price (1000)
\$4.82	\$4.09

VIII.2. Intrumentation amplifier (IA)

Ex: LT167 – Single resistor gain, programmable, precision intrumentation amplifier



N8 PACKAGE
8-LEAD PDIP

S8 PACKAGE
8-LEAD PLASTIC SO

T_{JMAX} = 150°C, θ_{JA} = 130°C/W (N8)
T_{JMAX} = 150°C, θ_{JA} = 190°C/W (S8)

APPLICATIONS

- Bridge Amplifiers
- Strain Gauge Amplifiers
- Thermocouple Amplifiers
- Differential to Single-Ended Converters
- Medical Instrumentation

Price (1 - 99)	Price (1000)
$6.45	$5.55

FEATURES

- **Single Gain Set Resistor: G = 1 to 10,000**
- **Gain Error: G = 10, 0.08% Max**
- **Input Offset Voltage Drift: 0.3µV/°C Max**
- **Meets IEC 1000-4-2 Level 4 ESD Tests with Two External 5k Resistors**
- Gain Nonlinearity: G = 10, 10ppm Max
- Input Offset Voltage: G = 10, 60µV Max
- Input Bias Current: 350pA Max
- PSRR at G = 1: 105dB Min
- CMRR at G = 1: 90dB Min
- Supply Current: 1.3mA Max
- Wide Supply Range: ±2.3V to ±18V
- 1kHz Voltage Noise: $7.5nV/\sqrt{Hz}$
- 0.1Hz to 10Hz Noise: $0.28µV_{P-P}$
- Available in 8-Pin PDIP and SO Packages

Datasheet: http://cds.linear.com/docs/Datasheet/1167fc.pdf