Chapter 3: Methods of analysis

I. Introduction.

II. Nodal analysis.

III. Mesh analysis.

VI. Nodal versus mesh analysis.

I. Introduction

- In chapter 2, we have studied the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws).

- This chapter will apply these laws to develop 02 powerful techniques for circuit analysis:
  - Nodal analysis: Base on KCL
  - Mesh analysis: Base on KVL

- With the 02 techniques, we can analyze almost any circuit by obtaining a set of simultaneous equations that are the solved to obtain the required values of current or voltage (power, energy)

II. Nodal analysis

- Nodal analysis provides a general procedure for analyzing circuits using *node voltage as the circuit variables* → *known as the node – voltage method.*

- Choosing node voltages instead of element voltages is convenient and *reduces the number of equations*.

II.1. Nodal analysis without voltage sources

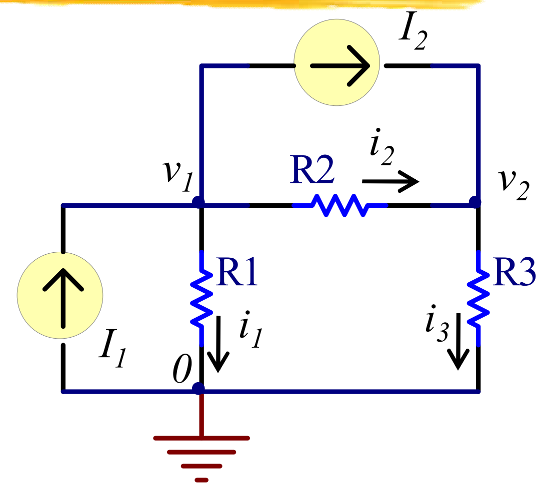
- Assuming that circuits with *n* nodes do not contain voltage sources

  - *Select a node as the reference node (ground, v = 0)*. Assign voltages v_1, v_2, … v_{n-1} to the remaining *n-1* nodes. All voltages are referenced to the reference node.

  - Apply *KCL to each of the n-1 non-reference nodes*. Use Ohm's law to *express the branch currents in terms of node voltages*.

  - Solve the resulting simultaneous equations to obtain the unknown node voltages.

II.1. Nodal analysis without voltage sources



Ex 1: Find the currents in this circuit.

- Choose node 0 as a reference node ($v_0 = 0$). Assign voltage of node *1* and node *2* with v_1 and v_2, respectively.
- Applying KCL to each non-reference node:
  - Add i_1, i_2, and i_3 as the currents on R_1, R_2, R_3
  - Applying KCL for node 1 and 2 gives: $\begin{cases} I_1 = I_2 + i_1 + i_2 \\ I_2 + i_2 = i_3 \end{cases}$
- Applying Ohm's law to express the currents in term of node voltages:

$$i_1 = \frac{v_1 - v_0}{R_1} = G_1 v_1 \qquad i_2 = \frac{v_1 - v_2}{R_2} = G_2(v_1 - v_2) \qquad i_3 = \frac{v_2 - v_0}{R_3} = G_3 v_2$$

$$\rightarrow \begin{cases} I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2) \\ I_2 + G_2(v_1 - v_2) = G_3 v_2 \end{cases} \rightarrow \begin{cases} (G_1 + G_2)v_1 - G_2 v_2 = I_1 - I_2 \\ -G_2 v_1 + (G_2 + G_3)v_2 = I_2 \end{cases}$$

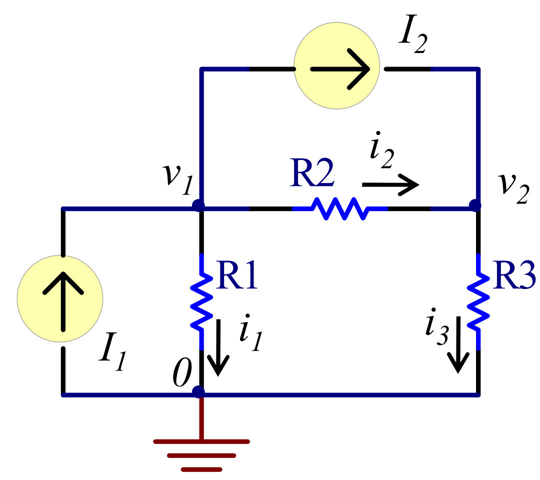
II.1. Nodal analysis without voltage sources

Ex 1: Find the currents in this circuit.

$$\rightarrow \begin{cases} (G_1 + G_2)v_1 - G_2 v_2 = I_1 - I_2 \\ -G_2 v_1 + (G_2 + G_3)v_2 = I_2 \end{cases} \leftrightarrow \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

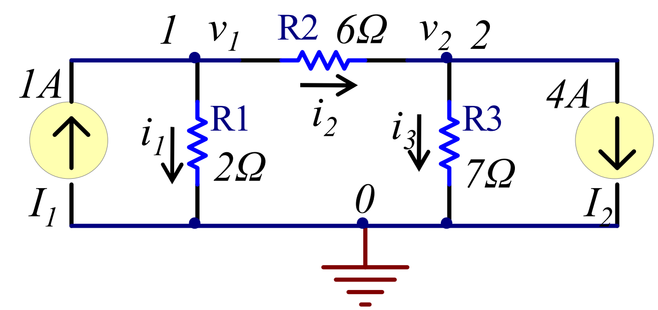
- We can obtain the node voltages v_1, v_2 using any standard method (*substitution method, elimination method, Cramer's rule, matrix inversion*) with software such as *Matlab*, *Mathcad, Maple, Quattro Pro*.

- After obtaining the node voltages, we can calculate the currents in circuit.



$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_2}{R_3}$$

II.1. Nodal analysis without voltage sources



Ex 2: Find the currents in this circuit.

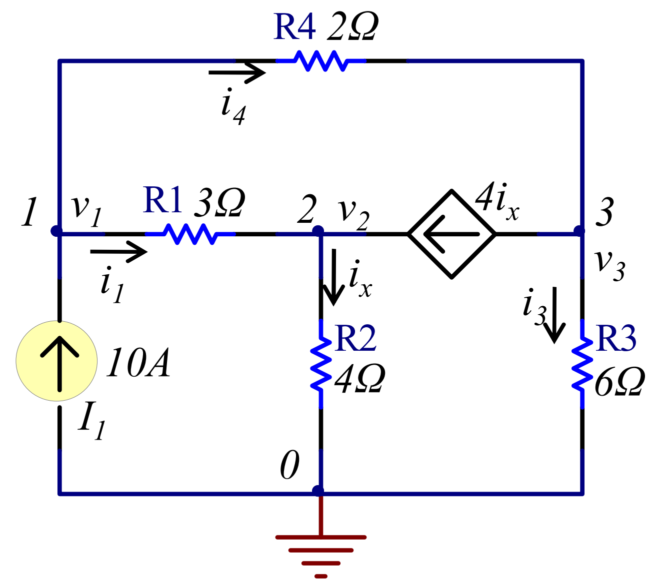
- Choose node 0 ~ reference node, node *1* ~ v_1, *node 2* ~ v_2.
- Applying KCL to each non-reference node: $\begin{cases} I_1 = i_1 + i_2 \\ i_2 = i_3 + I_2 \end{cases}$
- Applying Ohm's law: $i_1 = \dfrac{v_1}{R_1} = G_1 v_1$; $i_2 = \dfrac{v_1 - v_2}{R_2} = G_2(v_1 - v_2)$; $i_3 = \dfrac{v_2}{R_3} = G_3 v_2$

$$\to \begin{cases} (G_1 + G_2)v_1 - G_2 v_2 = I_1 \\ -G_2 v_1 + (G_2 + G_3)v_2 = -I_2 \end{cases} \to \begin{cases} 0{,}667 v_1 - 0{,}167 v_2 = 1 \\ -0{,}167 v_1 + 0{,}31 v_2 = -4 \end{cases} \to \begin{cases} v_1 = -2V \\ v_2 = -14V \end{cases}$$

- The currents in circuit: $i_1 = \dfrac{v_1}{R_1} = -1A$; $i_2 = \dfrac{v_1 - v_2}{R_2} = 2A$; $i_3 = \dfrac{v_2}{R_3} = -2A$

II.1. Nodal analysis without voltage sources

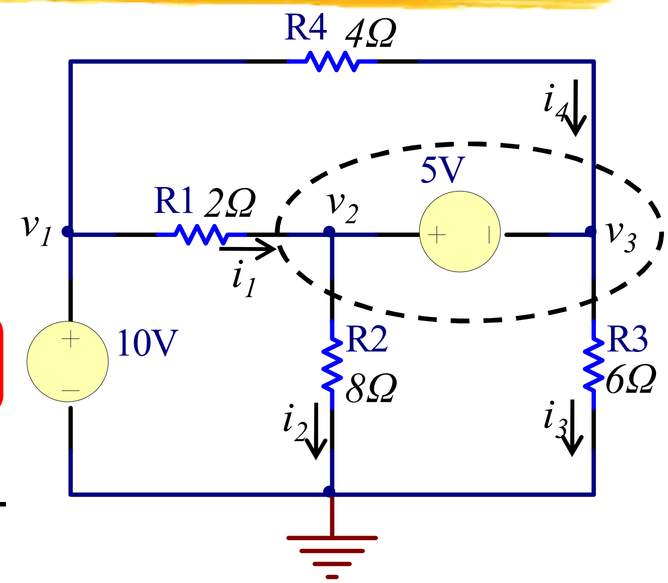
Ex 3: Find the voltages at the three non-reference nodes in the circuit.



- Choose node 0 ~ reference node, node *1* ~ v_1, *node 2* ~ v_2, node *3* ~ v_3.
- Applying KCL: $\begin{cases} i_1 + i_4 = I_1 \\ i_1 + 4i_x = i_x \\ i_4 = 4i_x + i_3 \end{cases}$
- Applying Ohm's law: $i_1 = G_1(v_1 - v_2)$; $i_x = G_2 v_2$; $i_3 = G_3 v_3$; $i_4 = G_4(v_1 - v_3)$

$$\rightarrow \begin{cases} (G_1 + G_4)v_1 - G_1 v_2 - G_4 v_3 = I_1 \\ -G_1 v_1 + (G_1 - 3G_2)v_2 = 0 \\ -G_4 v_1 + 4G_2 v_2 + (G_3 + G_4)v_3 = 0 \end{cases} \rightarrow \begin{cases} \frac{5}{6}v_1 - \frac{1}{3}v_2 - \frac{1}{2}v_3 = 10 \\ -\frac{1}{3}v_1 - \frac{5}{12}v_2 = 0 \\ -\frac{1}{2}v_1 + v_2 + \frac{2}{3}v_3 = 0 \end{cases} \rightarrow \begin{cases} v_1 = 80V \\ v_2 = -64V \\ v_3 = 156V \end{cases}$$

II.2. Nodal analysis with voltage sources



- We consider the following two possibilities:
  - Voltage source connects between reference node and non-reference node

    Voltage of non-reference node = voltage source

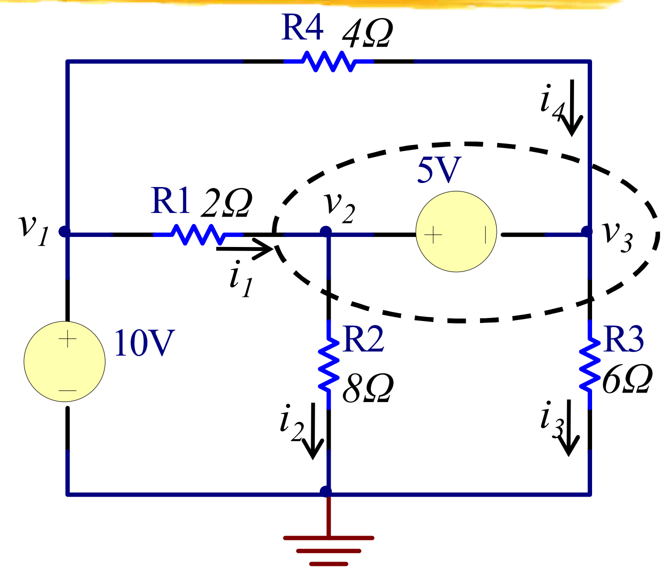
  - Voltage source connects between 2 non-reference nodes → form a *super-node*.

Super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

→Using the same three steps, except the super-node:

$$i_1 + i_4 = i_2 + i_3 \rightarrow \frac{v_1 - v_2}{R_1} + \frac{v_1 - v_3}{R_4} = \frac{v_2}{R_2} + \frac{v_3}{R_3} \qquad v_2 - v_3 = 5V$$

II.2. Nodal analysis with voltage sources

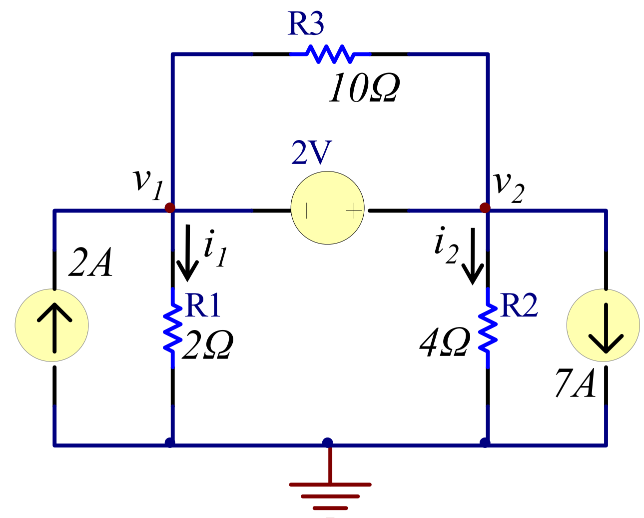


- We have a set of equations:

$$\begin{cases} v_1 = 10V \\ v_2 - v_3 = 5V \\ \dfrac{5}{8}v_2 + \dfrac{5}{12}v_3 = 7,5 \end{cases} \rightarrow \begin{cases} v_1 = 10V \\ v_2 = 9,2V \\ v_3 = 4,2V \end{cases}$$

- ***Note that:***
  - The voltage source inside the super-node provides a constraint equation needed to solve for the node voltages.
  - A super-node has no voltage of its own.
  - A super-node requires the application of both KCL and KVL.

II.2. Nodal analysis with voltage sources



Ex 1: Find the voltage node in this circuit using nodal analysis.

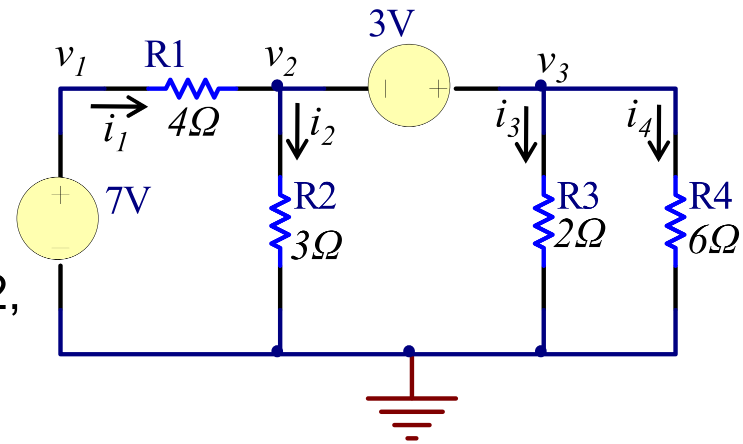
- Super-node includes the *2-V* source, node *1*, node *2* and R_3
- Applying KCL to the super-node gives:

$$2 = i_1 + i_2 + 7 \rightarrow 2 = \frac{v_1}{R_1} + \frac{v_2}{R_2} + 7 \leftrightarrow 2v_1 + v_2 = -20$$

- Applying KVL to the super-node gives: $v_2 = v_1 + 2$
- Solving the set of equations gives: $\begin{cases} v_1 = -7.33V \\ v_2 = -5,33V \end{cases}$

II.2. Nodal analysis with voltage sources

Ex 2: Find the voltage nodes and the currents in this circuit using nodal analysis.



- Super-node includes the *3-V* source, node 2, node *3.*

- Applying the KVL and KCL to the super-node gives a set of equations:

$$\begin{cases} \dfrac{v_1 - v_2}{R_1} = \dfrac{v_2}{R_2} + \dfrac{v_3}{R_3} + \dfrac{v_3}{R_4} \\ v_1 = 7V \\ v_3 = v_2 + 3 \end{cases} \rightarrow \begin{cases} \dfrac{7}{12} v_2 + \dfrac{2}{3} v_3 = \dfrac{7}{4} \\ v_1 = 7V \\ v_3 = v_2 + 3 \end{cases} \rightarrow \begin{cases} v_1 = 7V \\ v_2 = -0,2V \\ v_3 = 2,8V \end{cases}$$

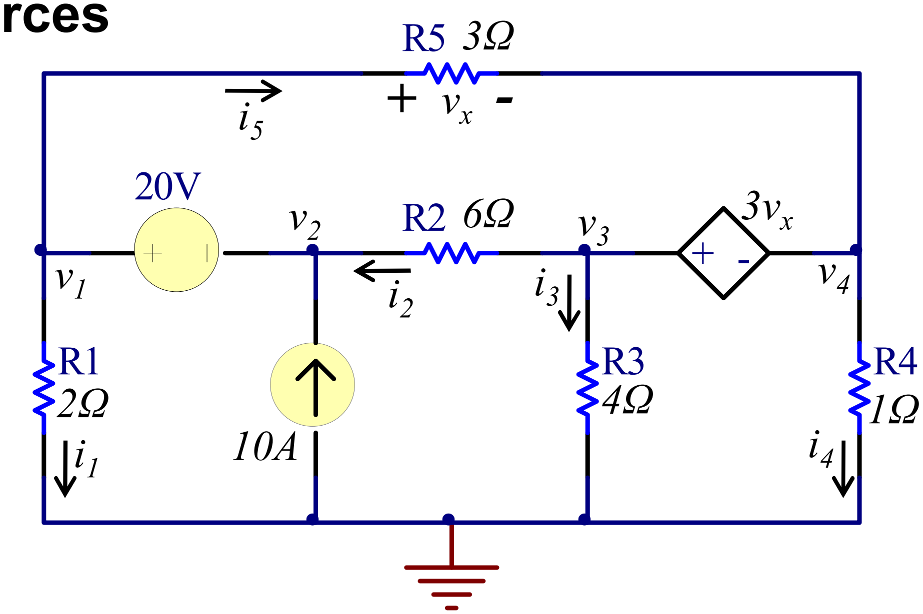
- Applying Ohm's law gives the currents:

$$i_1 = \frac{v_1 - v_2}{R_1} = 1,8A \qquad i_3 = \frac{v_3}{R_3} = 1,4A$$

$$i_2 = \frac{v_2}{R_2} = -0,067A \qquad i_4 = \frac{v_3}{R_4} = 0,467A$$

II.2. Nodal analysis with voltage sources

Ex 3: Find the voltage nodes in this circuit using nodal analysis.



- 1st super-node: Node 1 + node 2

$$i_1 + i_5 = i_2 + 10$$

$$\frac{v_1}{2} + \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + 10$$

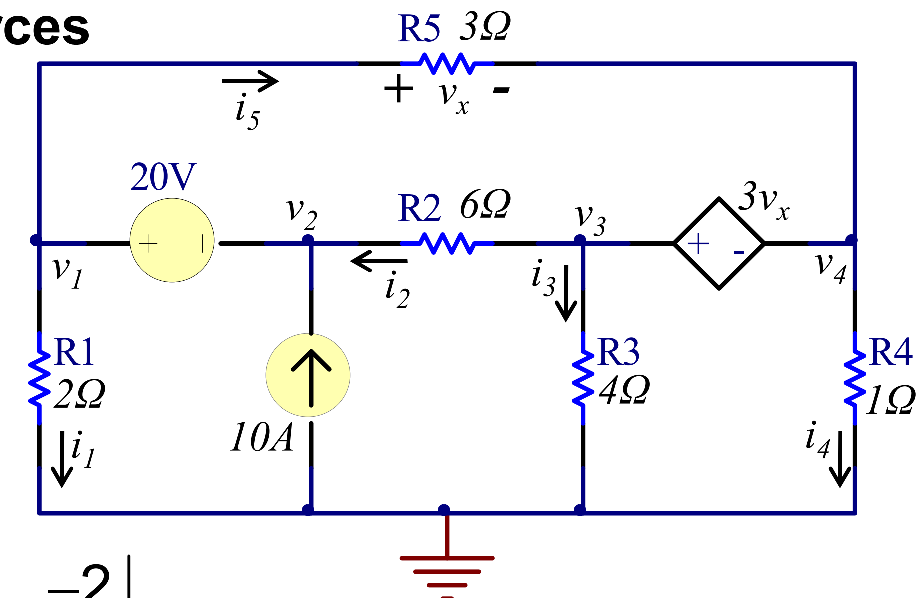
$$\boxed{5v_1 + v_2 - v_3 - 2v_4 = 60} \qquad \boxed{v_1 = 20 + v_2}$$

- 2nd super-node: Node 3 + node 4: $i_5 = i_2 + i_3 + i_4 \rightarrow \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_3}{4} + v_4$

$$\boxed{4v_1 + 2v_2 - 5v_3 - 16v_4 = 0} \quad v_3 = 3v_x + v_4 = 3(v_1 - v_4) + v_4 \rightarrow \boxed{3v_1 - v_3 - 2v_4 = 0}$$

II.2. Nodal analysis with voltage sources

Ex 3: Find the voltage nodes in this circuit using nodal analysis.



- We have a set of equations:

$$\begin{cases} 3v_1 - v_3 - 2v_4 = 0 \\ 6v_1 - v_3 - 2v_4 = 80 \\ 6v_1 - 5v_3 - 16v_4 = 40 \end{cases}$$

- Using Cramer's rule:

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18 \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120 \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

- The voltages gives:

$$v_1 = \frac{\Delta_1}{\Delta} = 26,67V$$

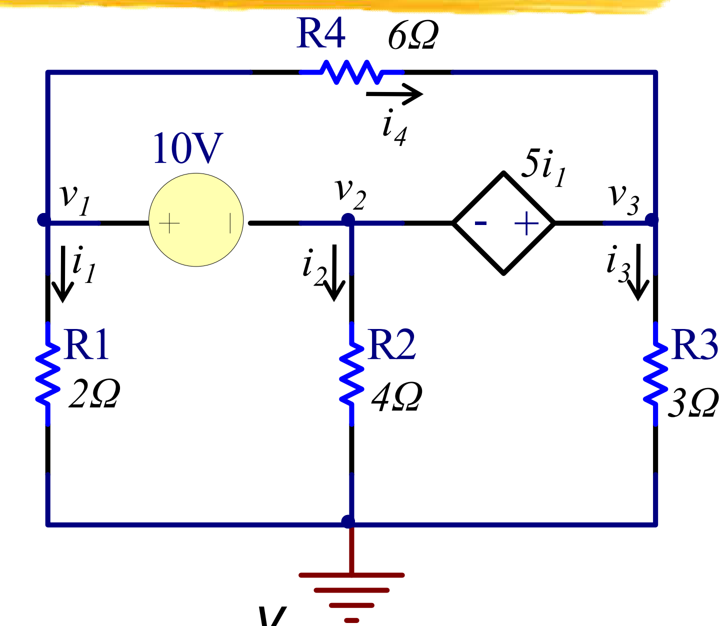
$$v_3 = \frac{\Delta_3}{\Delta} = 173.33V$$

$$v_4 = \frac{\Delta_4}{\Delta} = -46,67V$$

$$v_2 = v_1 - 20 = 6,67V$$

II.2. Nodal analysis with voltage sources

Ex 4: Find the voltage nodes and the current branches in this circuit using nodal analysis.



- 1st super-node contains: *10-V* source, $5i_1$ dependent source, and R_4

$$i_1 + i_2 + i_3 = 0 \rightarrow \frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0$$

$$v_1 - v_2 = 10$$

$$v_3 = 5i_1 + v_2 \quad \rightarrow 5v_1 + 2v_2 - 2v_3 = 0$$

- Solving the set of equations gives:

$$\begin{cases} v_1 = 3,043V \\ v_2 = -6,956V \\ v_3 = 0,652V \end{cases}$$

$$i_1 = \frac{v_1}{R_1} = 1,522A$$

$$i_2 = \frac{v_2}{R_2} = -1,739A$$

$$i_3 = \frac{v_3}{R_3} = 0,217A$$

$$i_4 = \frac{v_1 - v_3}{R_4} = 0,399A$$

II.3. Nodal analysis by inspection

- In general, if a circuit (with only *independent current sources*) has N non-reference nodes, the node-voltage equations can be written in terms of the conductances as:

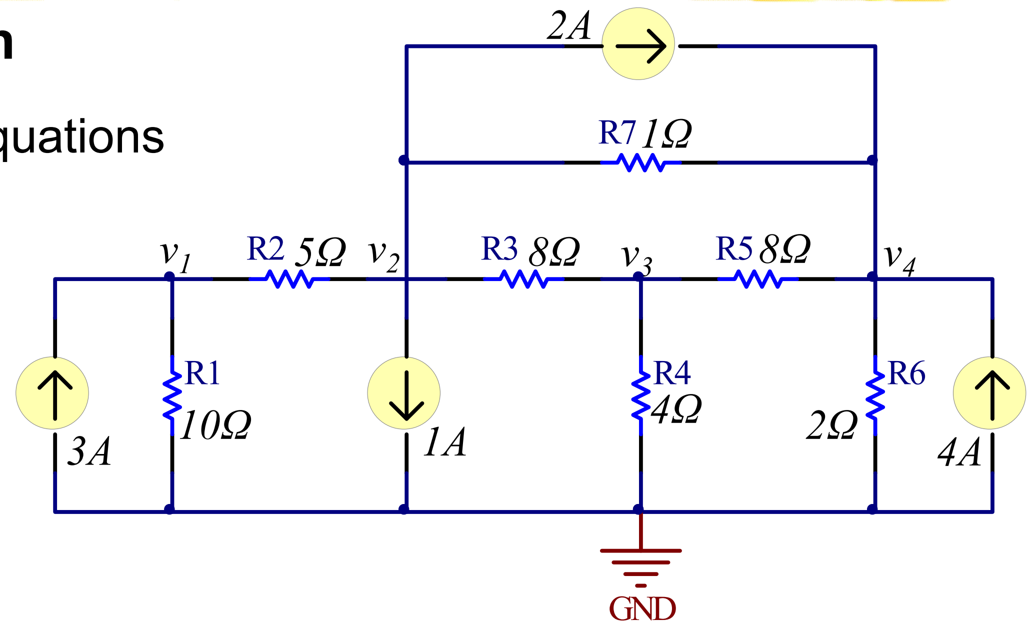
$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \quad \leftrightarrow \quad \mathbf{Gv} = \mathbf{i}$$

where:

- G_{kk} : Sum of the conductances connected to node k.
- $G_{kj} = G_{jk}$: Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$.
- v_k : Unknown voltage at node *k.*
- i_k : *Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive.*

II.3. Nodal analysis by inspection

Ex 5: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

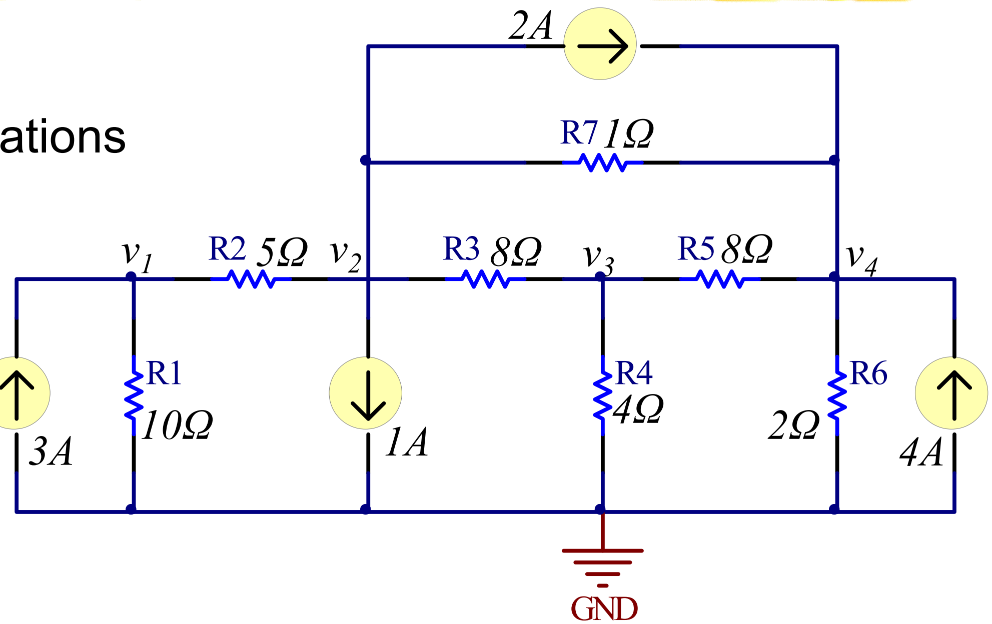
$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} = 0,3S \qquad G_{22} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_7} = 1,325S$$

$$G_{33} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = 0,5S \qquad G_{44} = \frac{1}{R_5} + \frac{1}{R_6} + \frac{1}{R_7} = 1,625S$$

II.3. Nodal analysis by inspection

Ex 5: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$G_{12} = G_{21} = -\frac{1}{R_2} = -0,2S$$

$$G_{34} = G_{43} = -\frac{1}{R_5} = -0,125S$$

$$G_{23} = G_{32} = -\frac{1}{R_3} = -0,125S$$

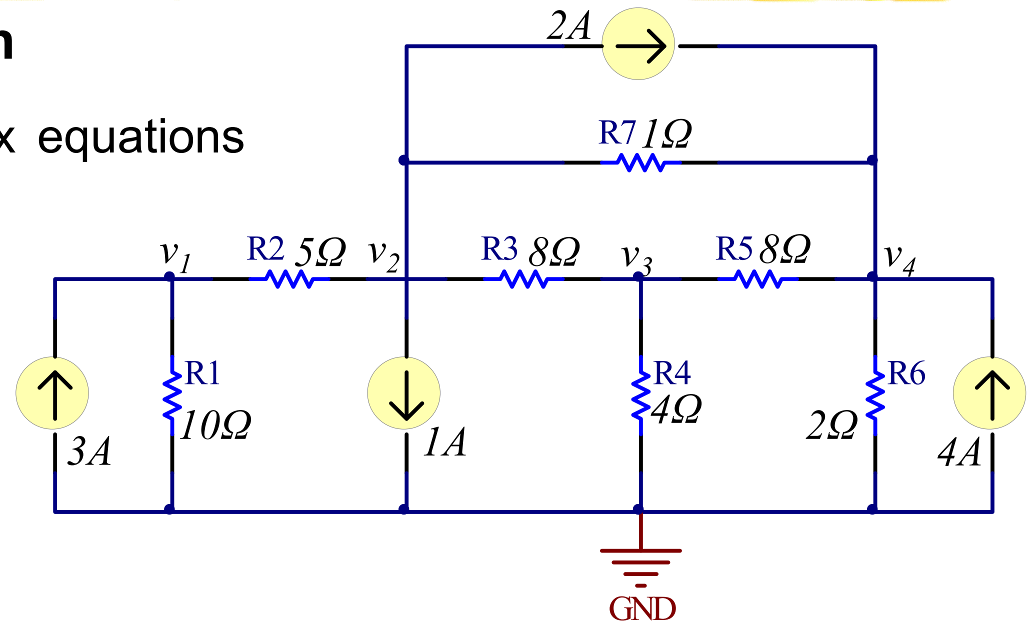
$$G_{13} = G_{31} = 0$$

$$G_{14} = G_{41} = 0$$

$$G_{24} = G_{42} = -\frac{1}{R_7} = -1S$$

II.3. Nodal analysis by inspection

Ex 3.5: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

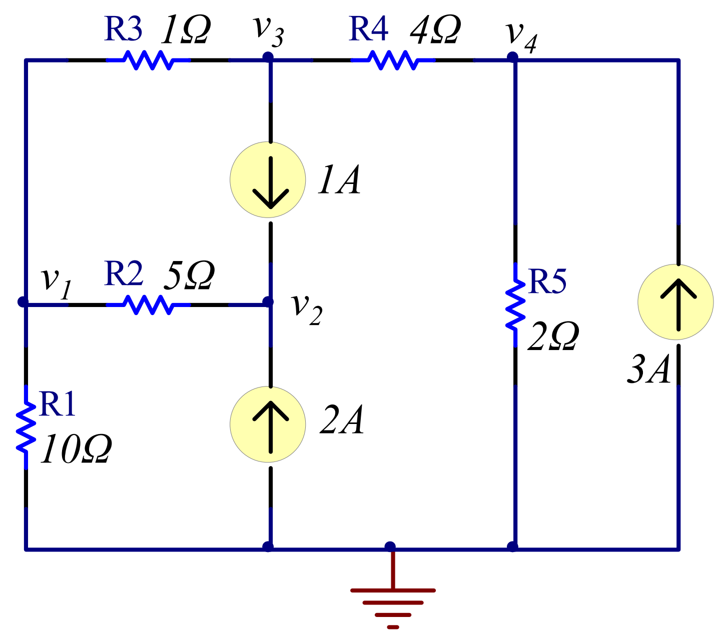
$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$i_1 = 3A$$
$$i_2 = -1 - 2 = -3A$$
$$i_3 = 0A$$
$$i_4 = 2 + 4 = 6A$$

$$\rightarrow \begin{bmatrix} 0,3 & -0,2 & 0 & 0 \\ -0,2 & 1,325 & -0,125 & -1 \\ 0 & -0,125 & 0,5 & -0,125 \\ 0 & -1 & -0,125 & 1,625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

II.3. Nodal analysis by inspection

Ex 3.6: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

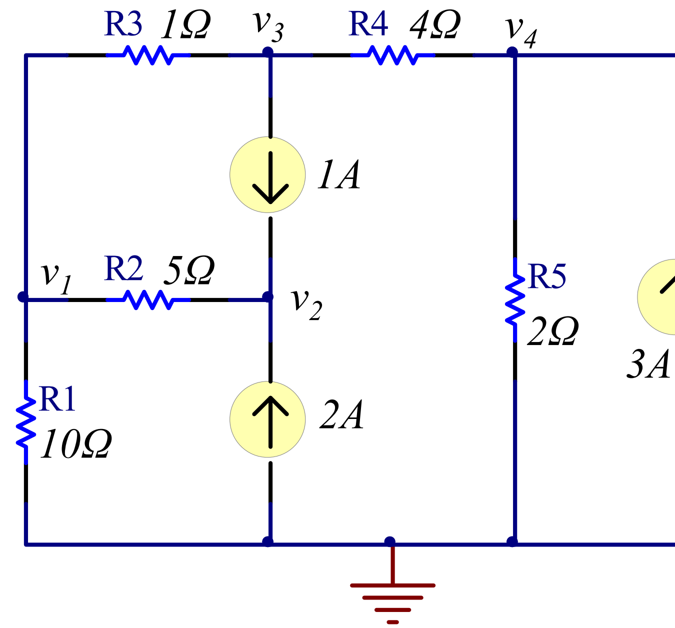
$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 1,3S \qquad G_{22} = \frac{1}{R_2} = 0,2S$$

$$G_{33} = \frac{1}{R_3} + \frac{1}{R_4} = 1,25S \qquad G_{44} = \frac{1}{R_4} + \frac{1}{R_5} = 0,75S$$

II.3. Nodal analysis by inspection

Ex 3.6: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

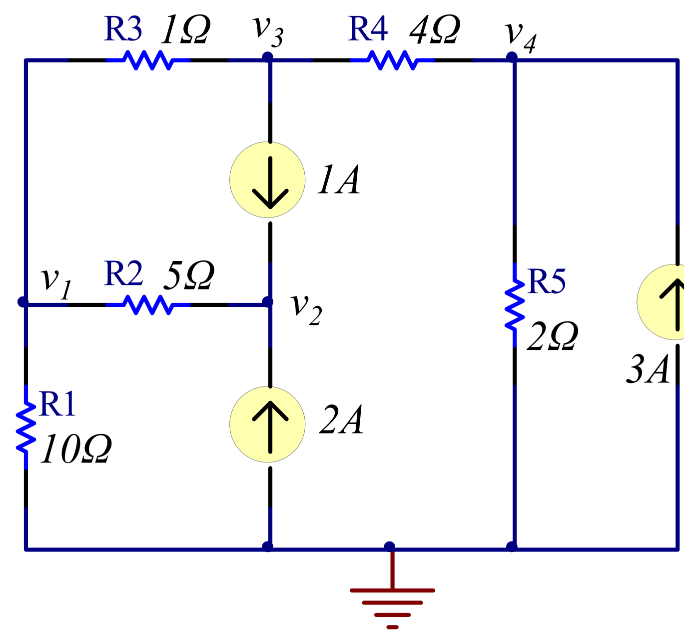
$$G_{12} = G_{21} = -\frac{1}{R_2} = -0,2S \qquad G_{23} = G_{32} = 0 \qquad G_{34} = G_{43} = -\frac{1}{R_4} = -0,25S$$

$$G_{13} = G_{31} = -\frac{1}{R_3} = -1S \qquad G_{24} = G_{42} = 0$$

$$G_{14} = G_{41} = 0$$

II.3. Nodal analysis by inspection

Ex 3.6: Write the node voltage matrix equations for this circuit.



- There are 4 non-reference nodes

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

$$i_1 = 0$$
$$i_2 = 1 + 2 = 3A$$
$$i_3 = -1A$$
$$i_4 = 3A$$

$$\rightarrow \begin{bmatrix} 1,3 & -0,2 & -1 & 0 \\ -0,2 & 0,2 & 0 & 0 \\ -1 & 0 & 1,25 & -0,25 \\ 0 & 0 & -0,25 & 0,75 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

III. Mesh analysis

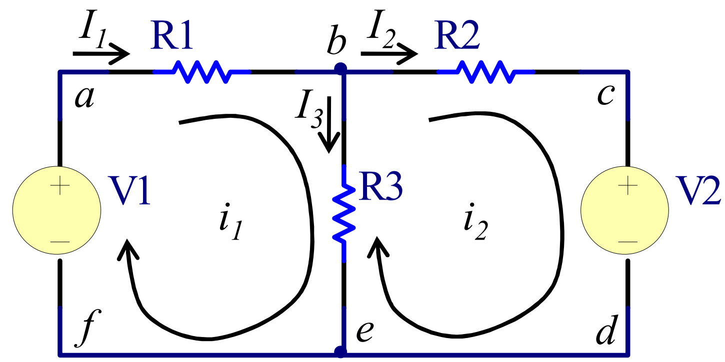
- Mesh analysis provides another general procedure for analyzing circuits, using *mesh current as the circuit variables* (known as loop analysis or the mesh current method).

- Using mesh current as the circuit variables is convenient and *reduces the number of equations*.

- A ***mesh*** is a loop that does not contain any other loop within it.

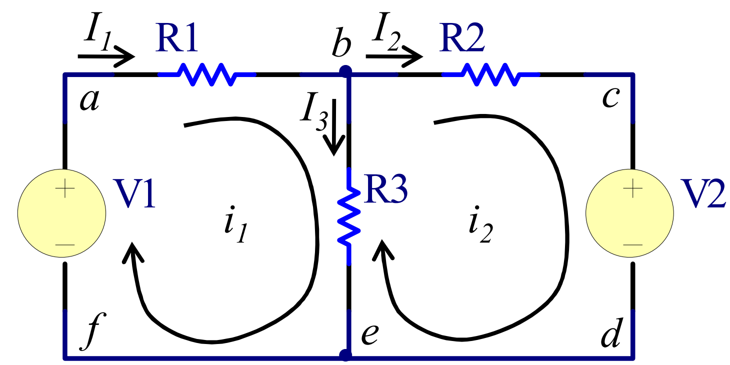
- Mesh analysis applies KVL to find unknown currents, and it is only applicable to a planar circuit.

III.1. Mesh analysis without current sources



- In this circuit, there are two meshes: *abefa*, and *bcdeb* (*abcdefa* is not a mesh)
- The current through a mesh is known as ***mesh current***.
- ***Steps to determine mesh currents:***
  - Assign mesh current $i_1, i_2, ..., i_n$ to the *n* meshes.
  - Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
  - Solve the resulting n equations to get the mesh currents.

III.1. Mesh analysis without current sources



- A mesh current may be assigned to each mesh (clockwise or counterclockwise)
- Applying KVL to mesh I, II gives:
  - Mesh I: $(R_1+R_3)i_1 - R_3 i_2 = V_1$
  - Mesh II: $-R_3 i_1 + (R_2+R_3)i_2 = -V_2$

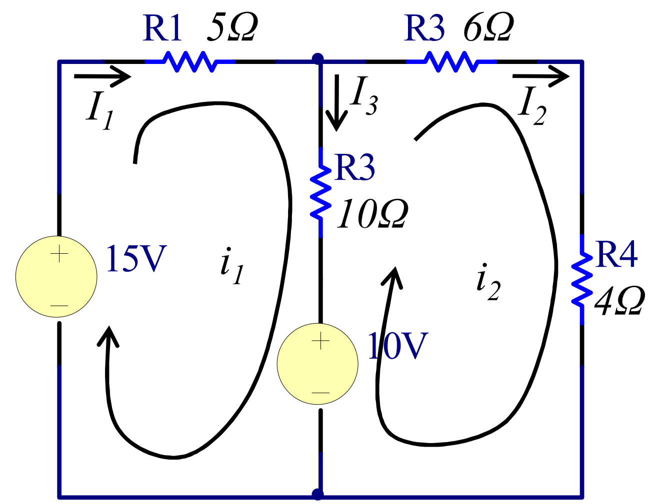
$$\rightarrow \begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

- After solving, we calculate the current elements: $I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$
- ***Notes:***
  - A circuit: *n* nodes, *b* branches, and *l* independent loops (mesh)

$$\boxed{l = b - n + 1}$$

  - The branch currents are different from the mesh currents unless the mesh is isolated.

III.1. Mesh analysis without current sources



Ex 1: Find the branch current I_1, I_2, I_3 using mesh analysis in this circuit.

- Applying KVL gives:
  - Mesh I: $-15+5i_1+10(i_1-i_2)+10=0$
  - Mesh II: $6i_2+4i_2+10(i_2-i_1)-10=0$

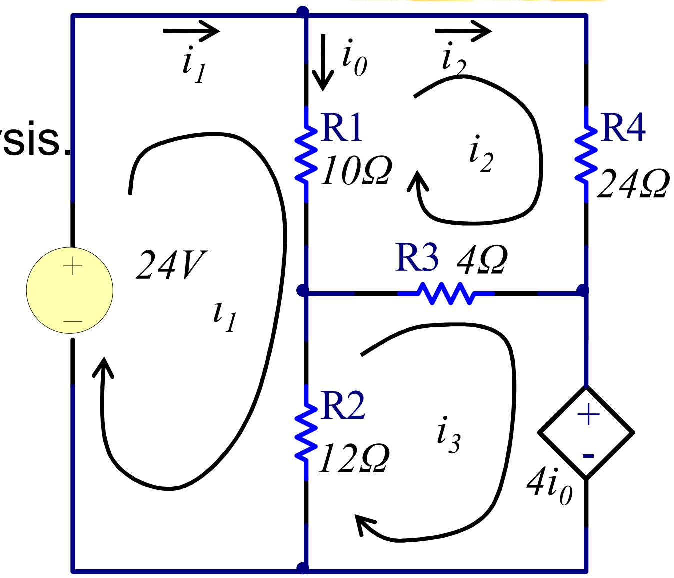
$$\rightarrow\begin{cases}3i_1-2i_2=1\\ i_1-2i_2=-1\end{cases}\qquad\rightarrow\begin{cases}i_1=1A\\ i_2=1A\end{cases}$$

- The current elements are:

$$I_1=i_1=1A\quad;\quad I_2=i_2=1A\quad;\quad I_3=i_1-i_2=0$$

III.1. Mesh analysis without current sources

Ex 2: Find the current i_0 in this circuit using mesh analysis.



- Applying KVL gives:
  - Mesh I: $-24+10(i_1-i_2)+12(i_1-i_3)=0$

    $$\boxed{11i_1-5i_2-6i_3=12}$$

  - Mesh II: $24i_2+4(i_2-i_3)+10(i_2-i_1)=0$

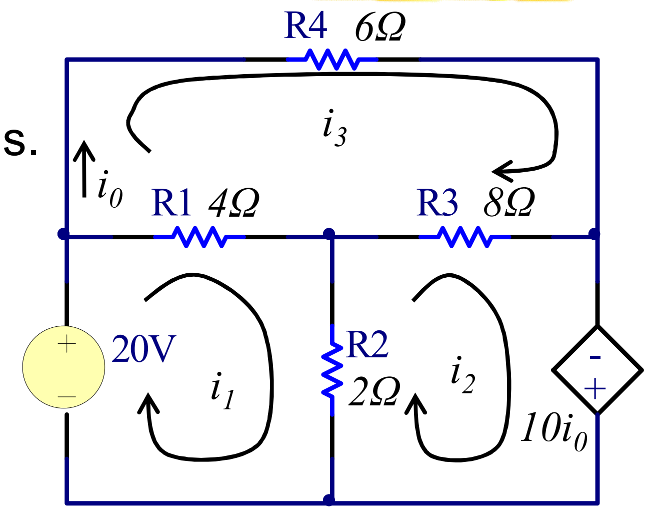
    $$\boxed{-5i_1+19i_2-2i_3=0}$$

  - Mesh III: $$\left.\begin{array}{r} 4i_0+12(i_3-i_1)+4(i_3-i_2)=0 \\ i_0=i_1-i_2 \end{array}\right\} \rightarrow \boxed{-i_1-i_2+2i_3=0}$$

- Then: $$\begin{cases} 11i_1-5i_2-6i_3=12 \\ -5i_1+19i_2-2i_3=0 \\ -i_1-i_2+2i_3=0 \end{cases} \rightarrow \begin{cases} i_1=2,25A \\ i_2=0,75A \\ i_3=1,5A \end{cases} \rightarrow i_0=i_1-i_2=1,5A$$

III.1. Mesh analysis without current sources

Ex 3: Find the current i_0 in this circuit using mesh analysis.



- Applying KVL gives:
  - Mesh I: $-20+4(i_1-i_3)+2(i_1-i_2)=0$

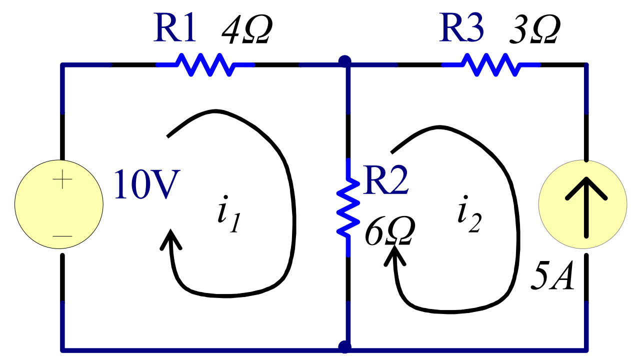
$$\boxed{6i_1-2i_2-4i_3=20}$$

  - Mesh II: $\left.\begin{array}{r} 2(i_2-i_1)+8(i_2-i_3)-10i_0=0 \\ i_0=i_3 \end{array}\right\} \rightarrow \boxed{-2i_1+10i_2-18i_3=0}$

  - Mesh III: $4(i_3-i_1)+8(i_3-i_2)+6i_3=0 \rightarrow \boxed{-4i_1-8i_2+18i_3=0}$

- Set of equations: $\begin{cases} 6i_1-2i_2-4i_3=20 \\ -2i_1+10i_2-18i_3=0 \\ -4i_1-8i_2+18i_3=0 \end{cases} \rightarrow \begin{cases} i_1=-3,21A \\ i_2=-9,64A \\ i_3=-5A \end{cases} \rightarrow i_0=-5A$

III.2. Mesh analysis with current sources



- In general, the presence of the current sources reduces the number of equations in the mesh analysis.
- Consider two cases:
  - ❖ A current source exists only in one mesh → mesh current = current source

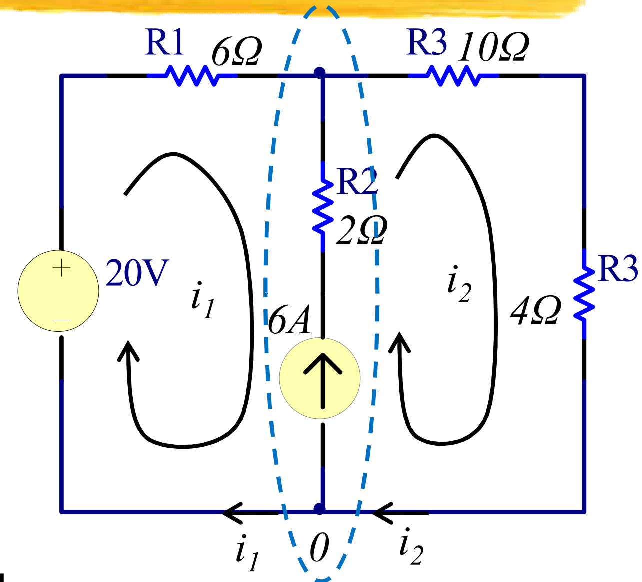
$$\begin{cases} 4i_1 + 6(i_1 - i_2) = 10 \\ i_2 = -5A \end{cases} \rightarrow \begin{cases} i_1 = -2A \\ i_2 = -5A \end{cases}$$

  - ❖ A current source exists between two meshes → create a ***super-mesh*** by excluding the current source and any elements connected in series with it

A ***super-mesh*** results when two meshes have a (dependent or independent) current source in common.

III.2. Mesh analysis with current sources

Ex 1: Find the branch currents using mesh analysis.



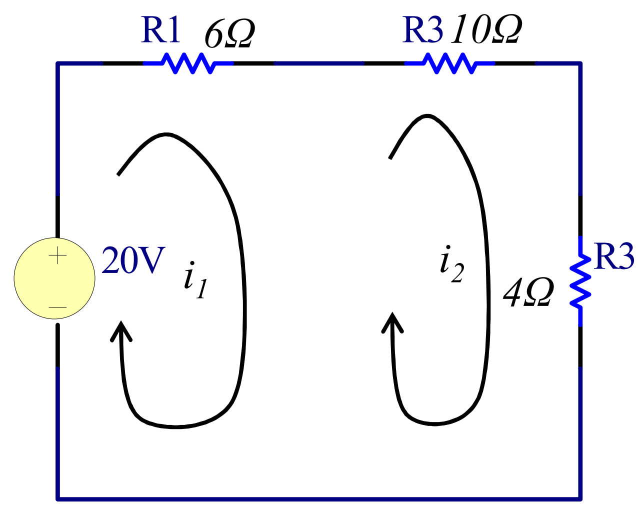
- There is a current source *6-A* between two mesh.
- Create a super-mesh.
- Applying KVL to the super-mesh gives:

$$-20+6i_1+10i_2+4i_2=0 \;\rightarrow 6i_1+14i_2=20$$

- Applying KCL to the node in the current source branch :

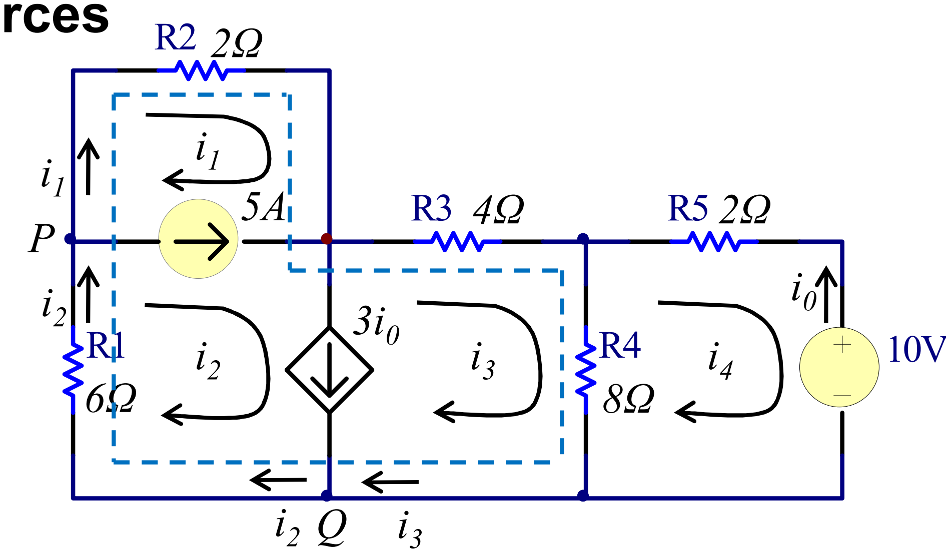
$$i_2=i_1+6$$

- ***Note that:***
  - The current source in the super-mesh provides the constraint equation to solve for the mesh currents
  - A super-mesh has no current of its own
  - A super-mesh requires the using of both KVL and KCL



III.2. Mesh analysis with current sources

Ex 2: Find i_1, i_4 using mesh analysis.



- There are two super-meshes
- Two super-meshes intersect → form a larger super-mesh
- Applying KVL to the larger super-mesh:

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$\rightarrow \boxed{i_1 + 3i_2 + 6i_3 - 4i_4 = 0}$$

- Applying KCL to the node *P*: $\boxed{i_2 = i_1 + 5}$
- Applying KCL to the node Q: $i_2 = i_3 + 3i_0 \xrightarrow{i_0 = -i_4} \boxed{i_2 = i_3 - 3i_4}$
- Applying KVL in mesh 4: $2i_4 + 8(i_4 - i_3) + 10 = 0 \rightarrow \boxed{5i_4 - 4i_3 = -5}$

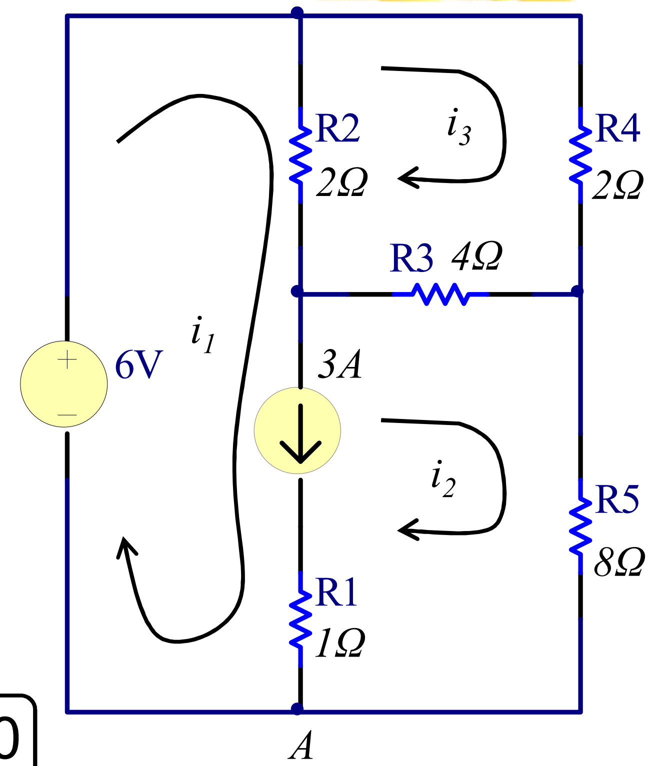
- These equations give:

$$i_1 = -7.5A \quad i_2 = -2.5A$$

$$i_3 = 3{,}93A \quad i_4 = 2.14A$$

III.2. Mesh analysis with current sources

Ex 3.3: Find i_1, i_2, i_3 using mesh analysis.



- Applying KVL to super-mesh:

$$-6+2(i_1-i_3)+4(i_2-i_3)+8i_2=0$$

$$\rightarrow \boxed{2i_1+12i_2-6i_3=6}$$

- Applying KCL to node A gives: $\boxed{i_1-i_2=3}$

- Applying KCL to the mesh III:

$$2(i_3-i_1)+4(i_3-i_2)+2i_3=0 \rightarrow \boxed{-2i_1-4i_2+8i_3=0}$$

- We have a set of equations:

$$\begin{cases} 2i_1+12i_2-6i_3=6 \\ i_1-i_2=3 \\ -2i_1-4i_2+8i_3=0 \end{cases} \rightarrow \begin{cases} i_1=3,47A \\ i_2=0,47A \\ i_3=1,11A \end{cases}$$

III.3. Mesh analysis by inspection

- In general, if a circuit (with only *independent voltage sources*) has N meshes, the mesh current equations can be expressed in terms of the resistances as:

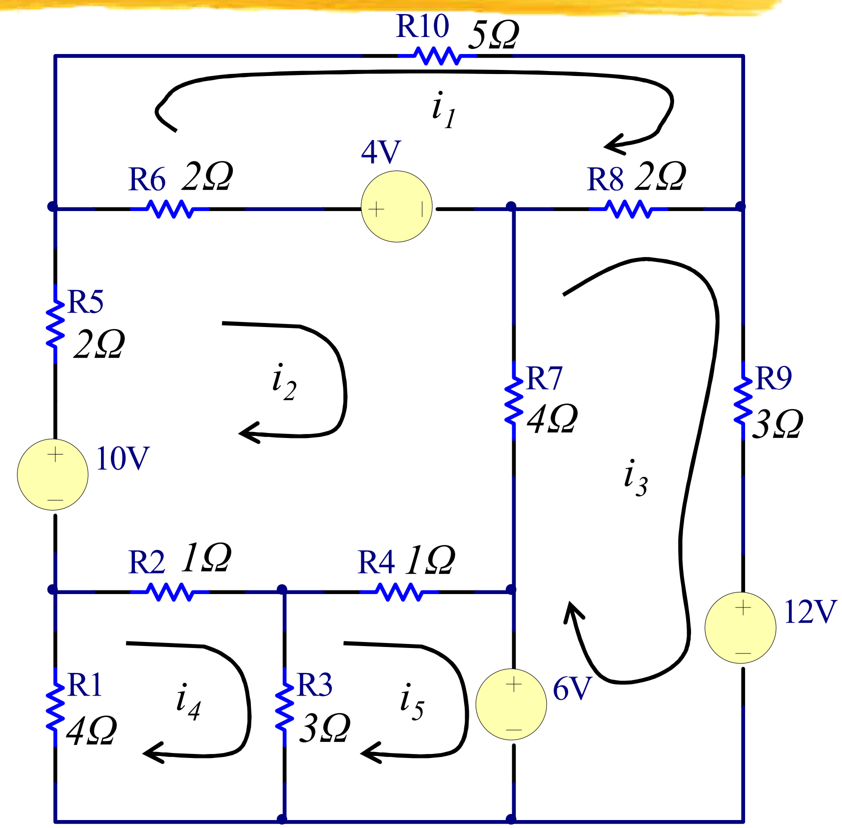
$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \quad \leftrightarrow \quad R\mathbf{i} = \mathbf{v}$$

where:

- R_{kk} : Sum of the resistances in mesh k.
- $R_{kj} = R_{jk}$: Negative of the sum of the resistances in common with meshes k and j, $k \neq j$.
- i_k : Unknown mesh current for mesh k in the clockwise direction.
- v_k : Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive.

III.3. Mesh analysis by inspection

Ex 3.4: Write the mesh current equations.



➢ There are 5 meshes

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$R_{11} = R_6 + R_8 + R_{10} = 9\Omega$$

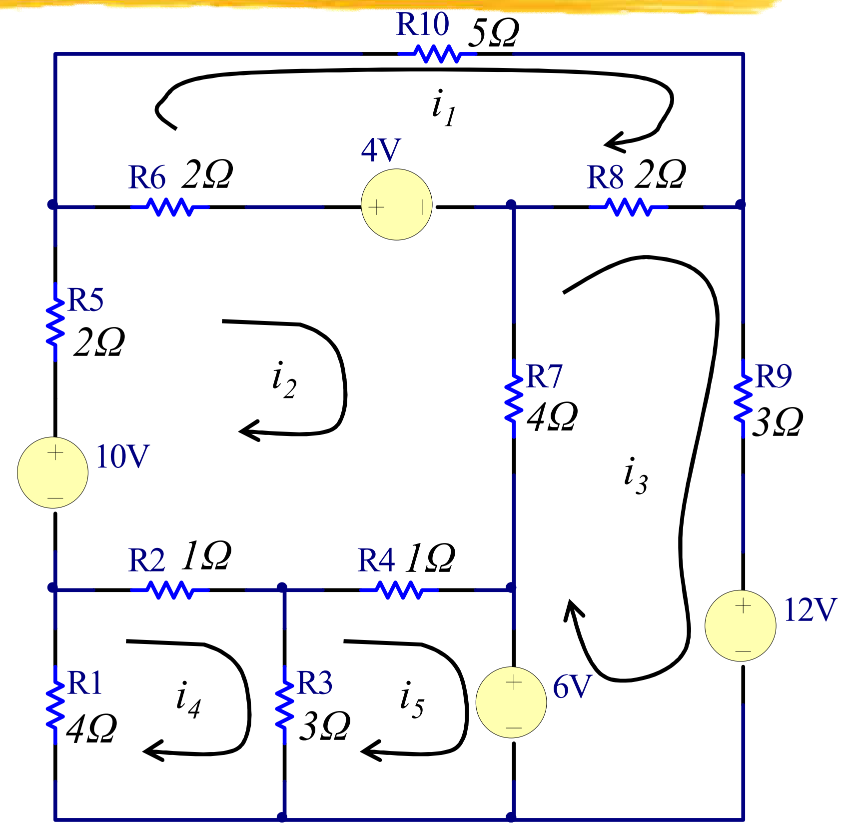
$$R_{22} = R_2 + R_4 + R_5 + R_6 + R_7 = 10\Omega$$

$$R_{33} = R_7 + R_8 + R_9 = 9\Omega \qquad R_{44} = R_1 + R_2 + R_3 = 8\Omega \qquad R_{55} = R_3 + R_4 = 4\Omega$$

III.3. Mesh analysis by inspection

Ex 3.4: Write the mesh current equations.

➢ There are 5 meshes



$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$R_{12} = R_{21} = -R_6 = -2\Omega$$

$$R_{13} = R_{31} = -R_8 = -2\Omega \qquad R_{23} = R_{32} = -R_7 = -4\Omega \qquad R_{34} = R_{43} = 0$$

$$R_{14} = R_{41} = 0 \qquad R_{24} = R_{42} = -R_2 = -1\Omega \qquad R_{35} = R_{53} = 0$$

$$R_{15} = R_{51} = 0 \qquad R_{25} = R_{52} = -R_4 = -1\Omega \qquad R_{45} = R_{54} = -R_3 = -3\Omega$$

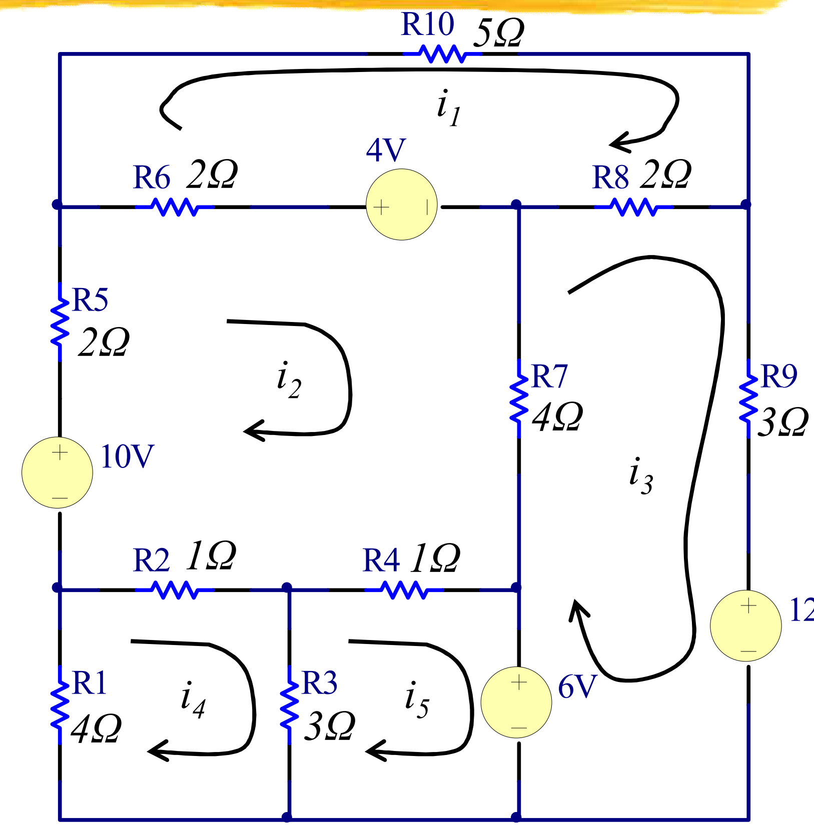
III.3. Mesh analysis by inspection

Ex 3.4: Write the mesh current equations.

- There are 5 meshes

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$



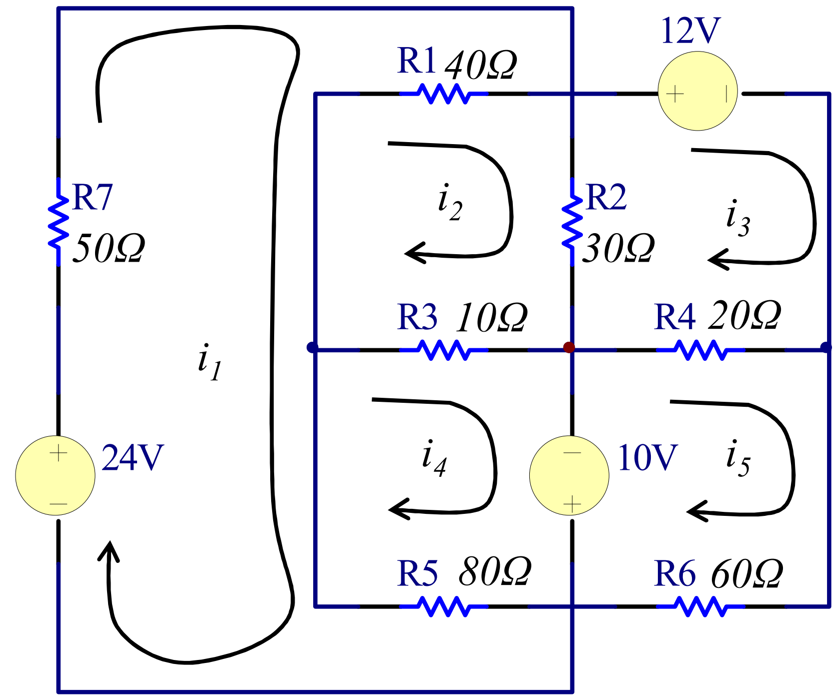
$$v_1 = 4V \qquad v_4 = 0$$

$$v_2 = 10 - 4 = 6V \qquad v_5 = -6V$$

$$v_3 = 6 - 12 = -6V$$

III.3. Mesh analysis by inspection

Ex 3.5: Write the mesh current equations.



- There are 5 meshes

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

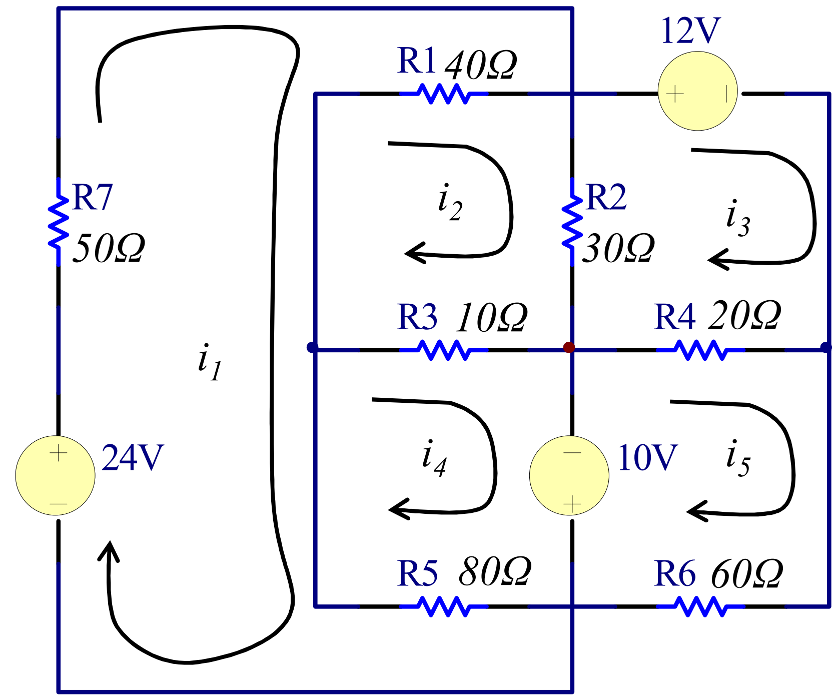
$$R_{11} = R_1 + R_5 + R_7 = 170\Omega$$

$$R_{22} = R_1 + R_2 + R_3 = 80\Omega$$

$$R_{33} = R_2 + R_4 = 50\Omega \qquad R_{44} = R_3 + R_5 = 90\Omega \qquad R_{55} = R_4 + R_6 = 80\Omega$$

III.3. Mesh analysis by inspection

Ex 3.5: Write the mesh current equations.



➢ There are 5 meshes

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$R_{12} = R_{21} = -R_1 = -40\Omega$$

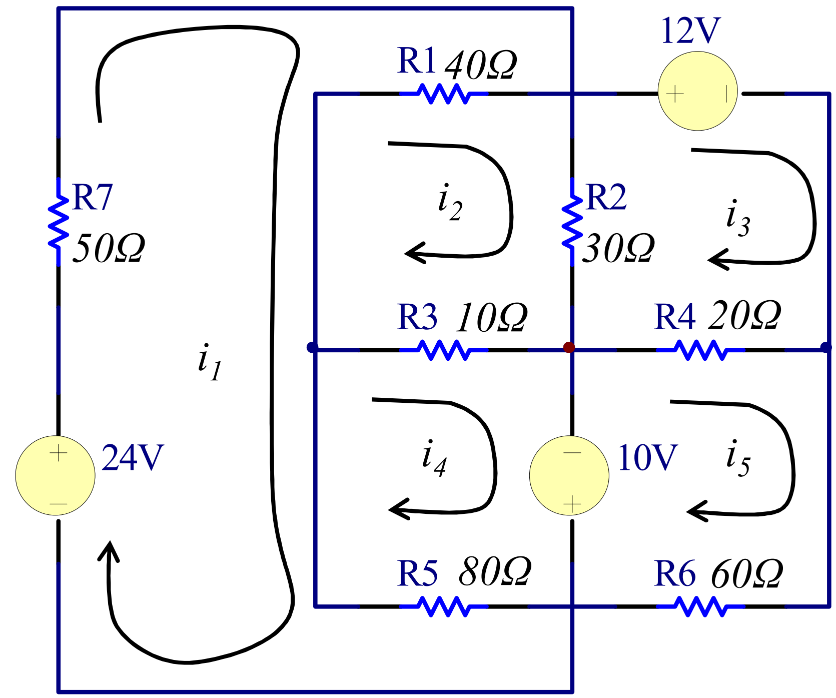
$$R_{13} = R_{31} = 0 \qquad R_{23} = R_{32} = -R_2 = -30\Omega \qquad R_{34} = R_{43} = 0$$

$$R_{14} = R_{41} = -R_5 = -80\Omega \qquad R_{24} = R_{42} = -R_3 = -10\Omega \qquad R_{35} = R_{53} = -R_4 = -20\Omega$$

$$R_{15} = R_{51} = 0 \qquad R_{25} = R_{52} = 0 \qquad R_{45} = R_{54} = 0$$

III.3. Mesh analysis by inspection

Ex 3.5: Write the mesh current equations.



- There are 5 meshes

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 170 & -40 & 0 & -80 & 0 \\ -40 & 80 & -30 & -10 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -10 & 0 & 90 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -12 \\ 10 \\ -10 \end{bmatrix}$$

$v_1 = 24V$ $\quad v_4 = 10V$

$v_2 = 0$ $\quad v_5 = -10V$

$v_3 = -12V$

IV. Nodal versus Mesh analysis

- Nodal and mesh analyses provide a systematic way of analyzing a complex network.

- ***Question: Given a network, which method is better or more efficient ?***

  - Select method that results in the *smaller number of equations*
    - ***Mesh analysis*** → many series-connected elements, voltage sources, or super-meshes; ***nodal analysis*** → parallel-connected elements, current sources, or super-nodes.
    - Circuits have $n < l$ → nodal analysis; but $l < n$ → mesh analysis.
  - *Information required:*
    - Node voltages are required → nodal analysis.
    - Branch or mesh currents are required → mesh analysis.
  - *Particular problems:* Analyzing transistor , op amp circuits, or non-planar circuit.